SHORT-TIME RADIATIVE TRANSFER THROUGH THIN SCATTERING SLABS : A NUMERICAL AND THEORETICAL STUDY

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INTRODUCTION

Radiative transfer through scattering media has attracted a great deal of interest recently, particularly for imaging applications through turbid media. Several techniques have been developed in order to determine the location of objects in strongly scattering biological tissues, using visible or near-infrared light[1]. Pulse transmission measurements on short time scales and optical coherence tomography give promising results[2]. Radiative (or conductive) transfer at short (time and length) scales is also a domain of growing interest with the rapid development of micro- and nano-technologies[3]. All these problems require a solution of the time-dependent radiative (or phonon) transfer equation in a scattering medium.

NUMERICAL STUDY

We have developed a numerical scheme to solve the time-dependent radiative transfer equation (RTE) in a scattering and absorbing medium. After a time-domain Fourier transform, the unsteady RTE takes the form of an equivalent steady-state RTE, involving an equivalent complex-valued extinction coefficient. This equation is solved numerically for a slab geometry, using a standard discrete-ordinate scheme[4]. An inverse Fourier transform leads to the time-dependent transmitted and reflected intensities. Examples of calculated transmitted pulses are shown in Fig. 1.

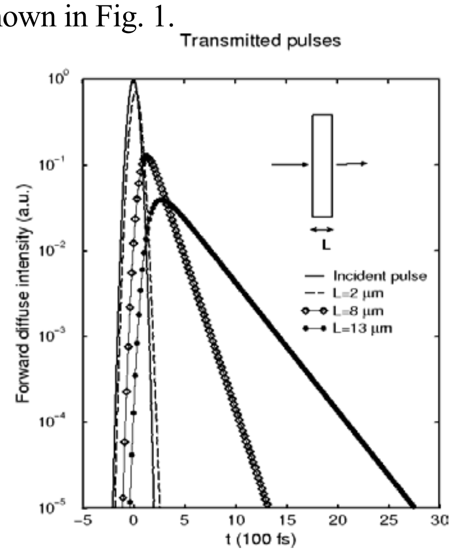

Figure 1 : transmitted diffuse intensity through a slab containing TiO_2 particles (radius 110 nm). Incident pulse : gaussian, with a width of 100 fs and a central wavelength λ=781 nm. When the slab thickness L is larger than 8 μm, the long-time exponential behavior of the pulse is a signature of the diffusive regime.

DEPENDENCE ON ABSORPTION AND DISPERSION OF THE DIFFUSION COEFFICIENT

The numerical solution of the RTE can be compared to approximate solutions, such as the diffusion approximation[5], which is widely used to describe light transport through tissues[1,2]. A peculiar point is that different derivations of the diffusion equation (starting from different transport theory) do not necessarily lead to the same expression for the diffusion coefficient D. In particular, the dependence on absorption may change. Nevertheless, the correct prediction of this dependence is a crucial issue for imaging applications[5].

Starting from the mode expansion of the specific intensity which is used in the discrete-ordinate scheme, we present a general method to define the diffusion coefficient, which is valid for any value of absorption and phase function. This asymptotic method is similar to that used in neutron transport theory[6]. We show that $D=D_{P1}\, g_0$, where D_{P1} is the diffusion coefficient obtained from the P_1 approximation of the RTE and g_0 is a correction factor. We compare this rigorously defined diffusion coefficient with coefficients obtained by other approaches, and show that it correctly accounts for the dependence on absorption and phase function. The influence for short scale transfer is discussed.

At short time scale, the diffusion approximation may be improved by completely accounting for the time-dependence of the radiative flux. Note that this amounts to use a telegrapher's equation, instead of the diffusion equation. In the frequency domain, the telegrapher's equation is equivalent to a diffusion equation with a dispersive (i.e. frequency dependent) diffusion coefficient. It can be solved as the classical diffusion equation. A Fourier transform leads to the time-dependent result, accounting for dispersion effects. For a slab geometry, we compare the results of the RTE, the diffusion approximation and the improved diffusion approximation (with a dispersive diffusion coefficient). We discuss the relevance of dispersion effects in the diffusion coefficient for short-scale radiative transfer.

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