EFFECT OF PARTICLES ON THE TURBULENCE INTENSITY OF A CARRIER PHASE FOR GAS-SOLID PIPE FLOW

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The paper presents the mathematical model describing the effects of the particles presence on the carrier gas turbulence generation and turbulence dissipation for pipe flow.

INTRODUCTION

The analysis of the effect of disperse phase on the characteristics of turbulence of a carrier flow is one of the main problem in the theory of two-phase turbulent flows¹⁻⁵. The effect of the particles presence on the turbulent flow structure is not single-valued, and, depending on the inertia, particles may have both laminarizing and turbulizing effect on the flow. The presence of relatively fine particles as a result of flow decelerating associated with their incomplete involvement in the gas fluctuation motion causes an additional dissipation and a decrease in the intensity of turbulent fluctuations. With a growth of the inertia of particles, the additional dissipation due to the interphase impulse exchange in the fluctuation motion decreases and becomes neglected small for large particles. It is the formation of a nonstationary vortex structure (turbulent wake) as a result of flow separation behind a large particle being flown about that must be regarded as the mechanism of turbulence generation. As a rule, as the inertia of particles growth, a tendency is observed toward the replacement of the laminarizing effect of the disperse phase by the turbulizing effect.

TURBULENCE DISSIPATION BY SMALL PARTICLES

The additional dissipation associated with the presence of small particles has the form

$$\varepsilon_p = 2M(k-k_i)/\tau_p, \qquad (1)$$

where M is the particles mass concentration, τ_p is the particles response time, $k = \langle u'_n u'_n \rangle/2$ is the turbulent energy of the carrier phase, $k_i = \langle u'_n v'_n \rangle/2$ is the kinetic energy of interphase interaction. Within the locally homogeneous approximation, the kinetic energy of interphase interaction, k_i , is directly related to the turbulent energy of carrier flow, k, by the relation

$$k_i = f_u k$$
, $f_u = \frac{1}{\tau_p} \int_0^\infty \Psi_u(\xi) \exp(-\xi/\tau_p) d\xi$, (2)

where Ψ_u is the autocorrelation function of gas velocity fluctuations along the particle trajectories. If the widely accepted exponential correlation

$$\Psi_u = \exp(-\xi/T_{Lp}),\tag{3}$$

is used as autocorrelation function, then the coefficient of involvement of particles in the fluctuation motion of carrier flow assumes the form

$$\int u = \frac{1 + \tau_n}{T_{In}}$$
(4)

Thus, the expression for additional dissipation (1) in view of (2) and (4) changes to

$$\varepsilon_p = \frac{2Mk}{T_{Lp}(1 + \tau_p / T_{Lp})} \quad . \tag{5}$$

Here, T_{Lp} is the time integral scale of gas fluctuation velocity, calculated along particle trajectory and characterizing the time of their interaction with energy-containing turbulent eddies of the carrier phase. For fine particles, T_{Lp} coincides with the integral Lagrangian scale of turbulence, T_L , defined by the relation

$$T_L = c_{\mu}^{1/2} k \, / \, \varepsilon \, , \, \varepsilon = c_{\mu}^{3/4} k^{3/2} \, / \, \ell \, , \qquad (6)$$

where $c_{\mu} = 0.09$, ε is the dissipation rate of turbulent energy of the carrier phase, ℓ is the mixing length by Prandtl-Nikuradze.

In view of the assumption made, (4)-(6) yield the following expression for the turbulence dissipation by the particles presence

$$\frac{\varepsilon_p}{\varepsilon} = \frac{2M}{C_{\mu}^{1/2}(1 + C_{\mu}^{1/4}\bar{\tau}_p\bar{k}^{1/2}/\bar{\ell})}$$

where $\bar{\tau}_p = \tau_p u_{*0} / R$, $\bar{k} = k / u_{*0}^2$, $\bar{\ell} = \ell / R$, u_{*0} is the friction velocity in the absence of particles, R is the pipe radius.

It follows from (7) that the additional dissipation increases as the mass concentration of particles increases and their inertia decreases. The effect of particles increases with the distance from the wall, which is explained by the decrease of their relative inertia with increasing time scale of turbulence.

TURBULENCE GENERATION BY LARGE PARTICLES

The main mechanism of turbulence generation due to the presence of large particles is the formation of nonstationary vortex structures caused by the separation of flow past these particles (aerodynamic wake behind the particles). It is well known, that one of the major shortcomings of most of the existing theoretical models for two-phase turbulent flows is the absence of the terms describing the generation of turbulence in the wake behind the particles for large particles Reynolds number. Therefore, such additional terms were introduced into the equation of the carrier phase turbulent energy balance using semi-empirical considerations.

Yarin and Hetsroni⁵ estimated the turbulization of flow by large particles using the self-similar solution for the far axisymmetric turbulent wake⁶. However, this approach is valid only for a very low volume concentration of the dispersed phase in the absence of the wake interference behind individual particles. In this study we will use the self-similar solution for a turbulent wake for the determination of additional turbulence generation in the fluctuation energy balance equation rather than for the direct calculation of the carrier flow turbulence characteristics. Such consideration of the self-similar solution for a far wake (i.e. the use of the solution in a local rather than integral sense) gives reason to hope that this model is valid for both low and moderate volume concentration of particles.

The distribution of the averaged velocity in a self-similar axisymmetric turbulent wake behind a body (particle) in flow is described as^6

$$\frac{U_{\delta} - U_x}{U_{\delta} - U_0} = f(\eta) \tag{8}$$

where
$$f(\eta) = (1 - \eta^{3/2})^2$$
, $\eta = y/\delta$, $\delta = 3(\beta^2 C_D d_p^2 / 16A)^{1/3} x^{1/3}$, $A = \int_0^{\infty} f(\eta) \eta d\eta = 0.13$,

$$U_{\delta} - U_0 = \frac{U_{\delta}}{9} (C_D d_p^2 / 16A\beta^4)^{1/3} x^{-2/3}.$$

Here, x and y are the coordinates in the direction along and normal to the main flow, U_0 is the velocity on the axis (y=0), U_{δ} is the velocity of unperturbed flow outside the wake, δ is the half width of the wake, d_p is the particle diameter, C_D is the particle aerodynamic drag coefficient. The constant β is determined as the ratio of the mixing length ℓ to the wake half width ($\ell = \beta \delta$) and is taken to be 0.2.

To calculate the turbulence characteristics of the flow in the wake, we used the turbulent energy balance equation in the diffusion-free approximation, i.e., under assumption that the generation is equal to dissipation. Received distribution of the turbulent energy generation over the cross section of the wake behind a particle we applied for calculation of the additional generation of turbulent energy in the volume of a cell occupied by one particle. The final expression for the additional generation of turbulence in the wake may be described as

$$P_p = a \left(\frac{C_D}{\beta}\right)^{4/3} \Phi \frac{W^3}{d_p},\tag{9}$$

where W = |U - V| is average velocity difference of the carrier and disperse phases, *a* constant determined by the following way

$$a = \frac{B}{27 \cdot 2^{11/3} A^{4/3}} = 0.027 , B = -\int_{0}^{1} f'^{3}(\eta) \eta d\eta = 0.6 , A = \int_{0}^{1} f(\eta) \eta d\eta = 0.13$$

As a value characterizing an influence of the turbulence additional generation in the particles wake may be used the relation between P_p and viscous dissipation ε

$$\frac{P_p}{\varepsilon} = \frac{a}{C_{\mu}^{3/4}} \left(\frac{C_D}{\beta}\right)^{4/3} \frac{\Phi \overline{W}^3 \overline{\ell}}{\overline{d}_p \overline{k}^{3/2}}$$

where $\overline{W} = W / u_{*0}$, $\overline{d}_p = d_p / R$.

From (11) it follows that the additional generation by particles presence depends on the volume, rather than mass (as in the case of a small particles), concentration of the dispersed phase. Because the value of W^3/d_p increases with particle size, the turbulizing effect of the dispersed phase on the flow also increases with its size. Moreover, according to (11), as a result of an increase of the mixing length, the particles effect increases with the distance from the wall. Therefore, similar to the case of small particles, the wall region is more conservative (less sensitive) than the flow core to the effect of dispersed phase on the turbulent structure of the carrier flow.

INFLUENCE OF THE PARTICLES ON TURBULENCE ENERGY

In order to analyze the effect of particles on the carrier gas turbulence intensity in a vertical pipe, we will use the equation of the turbulent energy balance

$$\frac{Dk}{Dt} = D + P - \varepsilon + P_p - \varepsilon_p .$$
⁽¹²⁾

The term on the left side of equation (12) describes the temporal variation of the turbulence energy and its convective transfer. The terms on the right side of the equation describe diffusion transfer, generation of turbulence from averaged motion due to the velocity gradient, viscous dissipation, additional generation of turbulence in the wake behind particles, and additional dissipation of turbulence by particles. For a steady-state hydrodynamically developed flow, the left side of equation (12) goes to zero. In order to obtain a simple analytical estimation of the effect of particles on fluid turbulence, the analysis is performed in the diffusion-free approximation, i.e., ignoring the contribution by the diffusion term in (12).

The turbulent energy generation in (12) caused by the averaged velocity shear is given by the equation

$$P = -\langle u'_{x}u'_{y} \rangle \frac{\partial U_{x}}{\partial y} = C_{\mu} \frac{k^{2}}{\varepsilon} \left(\frac{\partial U_{x}}{\partial y}\right)^{2}$$
(13)

where x and y are longitudinal and transverse (directed from wall to the pipe axis) coordinates.

With due regard for (4)-(6), (9), and (13), equation (12) yields the following expression for the turbulence energy of the carrier flow

$$k = \frac{\frac{\ell^2}{C_{\mu}^{1/2}} \left(\frac{\partial U_x}{\partial y}\right)^2 + \frac{a}{C_{\mu}^{3/4}} \left(\frac{C_D}{\beta}\right)^{4/3} \frac{\Phi W^3 \ell}{k^{1/2} d_p}}{1 + 2M f_u / C_{\mu}^{1/2}} \quad .$$
(14)

We assume that the effects of the particles on both the profile of the averaged gas velocity and the distribution of the mixing length are neglected small. In this way, expression (14) takes the form

$$\frac{k}{k_0} = \frac{1 + \frac{a}{C_{\mu}^{3/4}} \left(\frac{C_D}{\beta}\right)^{4/3} \frac{\Phi \overline{W}^3 \overline{\ell}}{\overline{k}_0^{3/2} \overline{d}_p} (k_0 / k)^{1/2}}{1 + \frac{2M}{C_{\mu}^{1/2} \left[1 + C_{\mu}^{1/4} (\overline{\tau}_p \overline{k}_0^{1/2} / \overline{\ell}) (k / k_0)^{1/2}\right]}}$$
(15)

Here $\bar{k}_0 = k_0 / u_{*0}^2$, k_0 is the turbulence energy in the absence of particles in the flow.

It can be seen, that (15) may describe both the attenuation and the augmentation of the carrier phase turbulence energy by the particles presence.



Fig.1 The effect of small particles on the turbulent energy of the carrier gas: (1) SiO_2 (50 µm), (2) Al_2O_3 (50 µm), (3) SiO_2 (100 µm), curve - expression (16).



Fig.2 The effect of large particles on the turbulent energy of the carrier gas: (1-3) plastic $(1420 \ \mu m)$, (4-6) plastic $(2780 \ \mu m)$, curve - expression (17).

For flow with small particles (15) yields to the following expression

$$\frac{k}{k_0} = \frac{1}{1 + 2X/C_{\mu}^{1/2}}, \quad X = \frac{M}{1 + C_{\mu}^{1/4} (\bar{\tau}_p \bar{k}_0^{1/2} / \bar{\ell}) (k/k_0)^{1/2}}$$
(16)

For flow with large particles (15) takes the form

$$\frac{k}{k_0} = 1 + \left(\frac{k_0}{k}\right)^{1/2} bY , \quad b = \frac{a}{C_{\mu}^{3/4} \beta^{4/3}} , \quad Y = \frac{C_D^{4/3} \Phi \overline{W}^3 \overline{\ell}}{\overline{d}_p \overline{k}_0^{3/2}} . \tag{17}$$

Figure 1 and figure 2 give the experimental data taken from works⁷⁻⁸ and generalized in the k/k_0 , X, and Y coordinates. It is seen that the data are fairly well generalized in these coordinates and are adequately described by expressions (16) and (17).

CONCLUSION

The mechanism of the carrier flow turbulence dissipation by small particles is studied. It was shown, that the laminarizing effect of disperse phase increases with the mass concentration of particles and the distance from the wall. As the inertia of particles increases (in the range being treated), their effect on turbulence decreases.

Based on the self-similar solution obtained for a far axisymmetric turbulent wake behind a body in flow, a model of generation of turbulence by large particles in a gas-solid flow is suggested. The model describes adequately the effect of turbulization of flow in a vertical pipe due to the formation of a wake behind large particles. The turbulizing effect of the dispersed phase increases with the volume concentration and size of particles, as well as with distance from wall.

Therefore, developed model takes into account both the turbulence attenuation and the turbulence augmentation phenomena in gas-solid flows. An adequate agreement is observed between the experimentally obtained and calculated results.

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