THE GEOMETRY OF INTERFACIAL WAVES IN VERTICAL CORE-ANNULAR FLOW

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INTRODUCTION

The annular pattern formed by two immiscible liquids of very different viscosities flowing together in a pipe, where the thicker fluid is placed at the center and the other one occupies the annulus (also called 'core annular flow') has found important applications in the transportation of viscous oils in horizontal pipes, through the addition of small quantities of a thinner fluid (usually water). In this work the vertical flow is focused, in view of its possible application in heavy oil production.

One of the rare experimental studies in vertical core flow was accomplished by Bai *et al.*[1]. Using a heavy oil and water at different flow rate combinations, they took photographs and recorded movies of the interfacial waves, observing the axi-symmetry of the flow. They also measured the speed and length of the interfacial waves as well as the pressure drop. Using their wave speed data, Bannwart[2] developed a correlation for the oil volume fraction which takes into account the density difference and the viscosity of the annulus fluid. Vanegas-Prada[2] measured the pressure drop in upward vertical core flow in a galvanized steel pipe, using a fuel oil (17,600 cP) and water. He verified that the frictional pressure drop was reduced, in average, 1200 times in comparison single phase oil flow, while the total pressure gradient was reduced about 100 times, which is a quite significant result in terms of oil production. He also developed a semi-empirical correlation for the pressure gradient, obtaining very satisfactory results.

The present work focus on the hydrodynamics of ascending vertical core flow for application in the design of production lines of heavy crudes. The approach followed herein is a phenomenological one, i.e. based directly on the governing equations of each phase as the method to determine the interface. The prominent role played by interfacial tension on the interface shape is emphasized.

LAPLACE-YOUNG EQUATION

In order to simplify the analysis of core-annular flow we assume: a) the flow is laminar incompressible isothermal and fluids are Newtonian; b) the fluid in the center (phase “1”) is much thicker than the annulus (phase “2”); and c) thin annulus. These assumptions allow to consider the core as a slug moving at a single velocity.

The Navier-Stokes equations describing the annulus flow are simplified and the Laplace-Young law is applied at the liquid-liquid interface. First, it is shown from these equations that the interface is circular in the pipe cross section, as expected. Besides, if inertial terms are neglected as in the *lubrication theory*[4], then it can be proved that the interface profile is described by

$$\frac{1}{r_i}\frac{d}{dr_i}\left(\frac{r_i}{\sqrt{1+r_i'^2}}\right)=C_2 \tag{1}$$

where $r_i = r_i(z)$ is the radial coordinate of the interface and $r_i' = \frac{dr_i}{dz}$. The above result is the Laplace-Young equation, which provides interfaces of periodic profile and constant mean curvature (C_2). Integration for the conditions

$$z=0 \Rightarrow \begin{cases} r_i = R_o \\ r_i' = 0 \\ \dfrac{r_i''}{\left(1+r_i'^2\right)^{3/2}} = \dfrac{k}{R_o} \end{cases} \tag{2}$$

leads to

$$\tilde{z}(\tilde{r}_i)=\int^{\tilde{r}_i}\frac{d\tilde{r}}{\sqrt{\dfrac{\tilde{r}}{\frac{(1-k)}{2}\left(\tilde{r}^2-1\right)+1}-1}} \tag{3}$$

where $\tilde{r}_i = \frac{r_i}{R_o}$ and $\tilde{z} = \frac{z}{R_o}$. The curvature parameter k must be in the range $-1 \le k \le 1$ and the mean curvature is $C_2 = (1-k)/R_o$. For the special case when $k = 0$ the interface is cylindrical and no waves are formed, thus R_o can be determined from information on the oil volume fraction. For the typically observed wavy interfaces, $k \neq 0$ and the higher k the steeper the wave (see Figure 1). For $0 < k \le 1$, the solutions are similar to the "bamboo waves" observed by Bai *et al.*[1]. The situations when $k < 0$ seem to correspond to down flow.

CURVATURE AND WAVELENGTH PREDICTION

Using the wavelength (λ) data of Bai *et al.*[1] it was possible to adjust the interface curvature parameter (k) according to the above model. For this purpose, the correlation for the oil volume fraction (ε) proposed by Bannwart[2] was employed, since it was developed for the same system of Bai *et al.*[1]. For all flow rates reported, an average value $k = 0.229$ fitted all the wavelength data with minor variation.

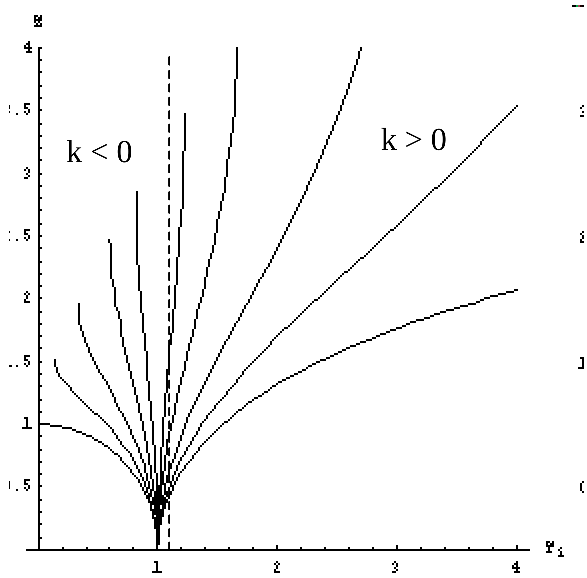

Figure 1. Dimensionless interface profile for several k values

The good quality of the fit can be observed in Figure 2.

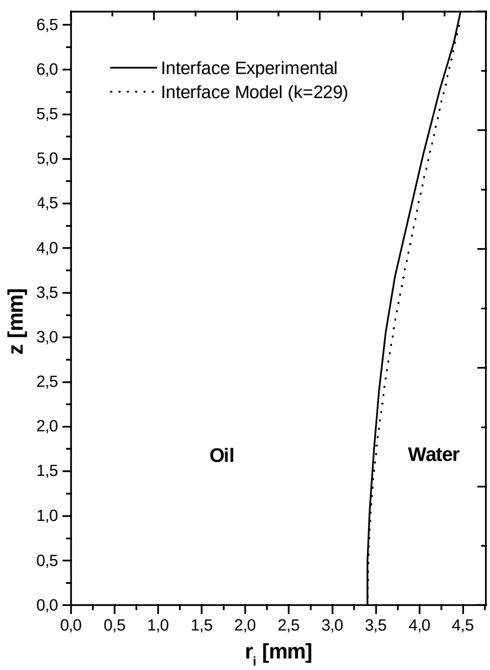

Figure 2. Comparison between the observed oil-water interface for $J_1 = 0.100$ m/s, $J_2 = 0.0468$ m/s (Bai *et al.*[1]) with the predicted by present theory for k = 0.229.

The wavelength was found to be reasonably well predicted by a constant Eötvos number of unity, i.e.

$$Eo_v = \frac{\frac{\pi}{4}\Delta\rho g \varepsilon \lambda D^2}{2\pi R_o \sigma} \cong \frac{\Delta\rho g \lambda D \sqrt{\varepsilon}}{4\sigma} \cong 1 \qquad (4)$$

as can be seen in Figure 3. Though a certain spread is observed, the results are supportive of the idea of a unity Eötvos number, whose average was 0.918 and standard deviation 17 %. Equation (4) agrees with the experimentally observed trend that λ decreases with increasing ε.

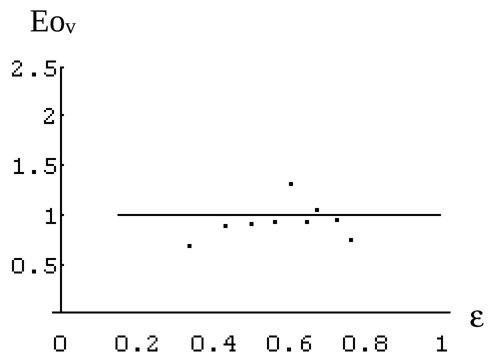

Figure 3. Eötvos number Eo_v versus oil volume fraction ε (using data from Bai *et al.*[1])

While the Laplace-Young equation describes the interface shape under lubrication theory assumptions only, it may be more generally valid in axi-symmetric liquid-liquid flows where inertia is present but interfacial tension produces no net force. The fact that the curvature parameter (k) remained constant for different flow rates indicates that inertia is probably unimportant in the experiments reported.

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