BUBBLE PLUMES AND THE COANDA EFFECT

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This paper experimentally describes the behaviour of a gas-liquid bubble plume set to develop adjacent to another plume. When this happens, the plume exhibits a type of a Coanda effect, bending towards the other plume. The local gas fraction measurements are carried out using the electro-resistivity probe technique in an air-water system. The deflection angle of the plumes is shown to present a logarithmic dependence on the modified Weber and Froude numbers of the flow. In addition, a simple theory based on integral methods is advanced for the prediction of the plume deflection; the theory considers a variable entrainment coefficient.

INTRODUCTION

The purpose of this work is to investigate experimentally the behaviour of bubble plumes that develop near to another bubble plume. Under this condition the plume is seen to develop not axisymmetrically, but deflected sideways. The emphasis here is on accounting for how the proximity of another plume deflects the plume, altering its mean properties. As a bonus, the paper develops a simple integral theory to describe the phenomenon. The theory is an extension of the theory of Ditmars and Cederwall[1], making use, for the sake of simplicity, of Gaussian profiles and of Boussinesq assumption. To measure the main characteristics of the plumes, an electro-resistivity probe system was used. This system was chosen for being simple but still capable of conveying many useful information on the problem; it will be discussed in detail in an appropriate section. Here, it suffices to say that by an analysis of the experimental data, the deflection of the plume has been evaluated as a function of several parameters of interest including the gas flow rate and the distance between the point sources.

THE COANDA EFFECT

The Coanda Effect is the tendency of a fluid, either gaseous or liquid, to cling to a vertical surface that is near to an orifice from which the fluid emerges (Reba[2]). An easily verified experimental fact is that when two point sources of buoyancy (or of momentum) are placed side by side, the two resulting turbulent plumes (or jets) tend to bent towards each other. In this case, the bending of the plumes is not motivated by a pressure difference, as observed when the resulting plumes are laminar (Pera[3]). When the flow is turbulent, the plumes bend due to a restriction in the entrainment of external fluid by the mean flow. Particularly in the axisymmetric geometry, the role played by pressure difference is negligible when compared with that of turbulent entrainment. Despite the difference in the nature of the forces that govern each phenomenon, some authors still consider both (pressure difference and entrainment restraint) as examples of the Coanda effect.

In the present work, the authors suggests that the bending of a turbulent round bubble plume is caused by a momentum flux unbalance related to the entrainment restraint due to the presence of an adjacent wall. The analysis in the subsequent sections of this paper will be entirely based on this premise.

EXPERIMENTS

The experimental technique

The working principle of the experimental technique is based on the difference between the electrical conductivity (resistivity) of the phases. Since the electrical conductivity of water is much higher than that of air, it is assumed, for practical purposes, that only the continuous phase (liquid) is capable of conducting an electrical current. Accordingly to Herringe[4], resistivity sensors are the most suitable technique for measurements in two-phase mixtures where the continuous phase is conductive. The main adversity of the technique is the existence of an in-stream sensor, which affects the structure of the flow. In a double channel system (whether AC or DC supply), the difference in electrical resistivity between the phases can be sensed by the electrodes in the two-phase flow so that parameters like the local time-averaged gas fraction, the rise velocity and the pierced length of bubbles can be obtained through the analysis of the output signal.

The measuring system used in this work comprises a signal conditioning module and five double channel needle probes. The signal conditioning module is composed of five identical electronic circuits (one for each double channel) connected to the same electrical reference (that can be either the probe sheath or a third electrode immersed in the single phase region). The circuits are fed by a 12V DC signal and their main component is the monolithic bipolar integrated circuit LM1830N, which was devised for use in fluid measurement and detection systems. An AC supply system was chosen because it is superior to the DC method. DC systems are subject to problems such as polarisation and electrochemical attack.

A detailed description of the complete experimental apparatus can be found in Barbosa[5]. Basically, it comprises a water tank, an air injection system, a 2D traversing mechanism and a data acquisition and analysis system. The glass water tank has dimensions 1x1x1 meter and is filled with a 3 gram per litre sodium chloride solution (brine). The air injection system is composed of a mass flow meter and two 3.2 *mm* diameter hole injection nozzles. The data acquisition and analysis system consists of a microcomputer with an interface data acquisition board, an oscilloscope, a signal conditioner module and the electro-resistivity probe.

The probe is placed perpendicularly to the tank bottom. The water depth is kept constant and equal to 0.85*m*. Approximately 15 minutes of continuous gas flow are necessary to guarantee a steady-state condition. Measurements of bubble plume properties are made for several gas flow rate and injection nozzle conditions with the aid of the traversing mechanism. Table 1 summarises the experimental conditions examined.

Table 1
Experimental test conditions.

Case	s[cm]	q[l/min]	*Fr*	*We*
i	3.25	1.2	1.12e-3	0.2910
j	3.25	2.4	4.50e-3	1.1638
k	6.4	1.2	3.80e-5	0.0381
l	6.4	2.4	1.52e-4	0.1524
m	9.05	1.2	6.72e-6	0.0135
n	9.05	2.4	2.69e-5	0.0539

The gas fraction at a point in the flow is a time-averaged property given by,

$$c(r,x,t) = \frac{1}{T}\int_0^T I(r,x,t)\, dt \qquad (1)$$

where T is the total sampling time and I is the digital output signal from the conditioning module.

Results

The local void fraction profiles for several x-stations were measured. Figure 1 shows measurements of void fraction profiles taken in the plane defined by the centerline of the plumes. The results are presented for four stations (x = 200, 270, 350 and 420 *mm*, where x denotes the vertical distance from the gas point source). The profile corresponding to the lowest station presents well defined Gaussian curves showing that the plume interaction cannot still be felt at this height. All subsequent profiles, however, present a noticeable deformation due the proximity of the other plume. The points of maximum void fraction are seen to migrate to the centre of the geometrical arrangement, giving origin well downstream, to a single plume with a well defined shape. The points of maximum void fraction were also noted. They were taken as the reference point for the evaluation of the position of the centreline of the plume. From previous studies, we know these points to correspond to the points where the liquid phase velocity is maximum as well.

The deflection of the plume for several gas flow rates and distances from the wall was evaluated in terms of non-dimensional coordinates. Most of the trajectories can be reasonably approximated by a straight line. For the plumes whose distance from each other is large this trend is particularly well defined. We may thus construct graphs for the variation of the deflection angle (ϕ) defined by these straight lines as a function of Froude and Weber numbers; they show that ϕ increases logarithmically and monotonically with both Fr and We according to the two displayed curves. These curves can be approximated by straight lines.

The non-dimensional groups were defined accordingly to

$$F_r = \frac{q^2}{g\, s^5}\,, \qquad W_e = \frac{\Delta\rho\, q^2}{\sigma\, s^3} \qquad (2)$$

where g is the acceleration of gravity, q is the gas flow rate issuing from each source, σ is the surface tension and $\Delta\rho$ is the density difference between the phases. The quantities above are, respectively, the modified Froude, Fr, and Weber, We, numbers based on the halved distance between the sources, s or the distance from a wall.

THEORY

Mean-flow theories for bubble plumes are integral theories for which the forms of the radial distributions of the velocity and of the density deficiency between the plume and the surrounding fluid are considered to be known in advance. In fact, since the pioneering work of Kobus[6] to the more recent theories, very little in terms of the formulation has changed. The recent theories have incorporated many novelties, but the basic formulation of the problem remains the same; the governing equations are constructed from the conditions of conservation of gas, conservation of liquid, and the change of momentum flux with buoyancy.

In the present work, the theory of Ditmars and Cederwall[1] will be used as a basis for our developments. By allowing the entrainment coefficient to vary along the plume periphery, the effect of deflection provoked by an adjacent twin plume can be conveniently modelled. This a very simple premise, which yields very good results. Since the basic details of the theory can be found in Ditmars and Cederwall[1], only a brief mention of them will be made here. The velocity and mean density defect are given by

$$u(x,r)=u_m(x)\ e^{-r^2/b^2} \tag{3}$$

$$\rho_a-\rho_m(x)=\Delta\rho_m(x)\ e^{-r^2/(\lambda\, b)^2} \tag{4}$$

where b(x) is the lateral dimension of the plume, ρ_a is the water density and λ the lateral spread of density deficiency to momentum.

The liquid volume at any elevation and the momentum flux are given respectively by

$$Q=\int_0^\infty 2\pi\ u\ r\ dr=\pi\ u_m\ b^2=\pi\ u_m b^2\text{, and} \tag{5}$$

$$M=\int_0^\infty 2\pi\ \rho_a\ u^2\ r\ dr=\frac{\pi}{2}\rho_a u_m^2 b^2. \tag{6}$$

The entrainment assumption is now taken as the explicit starting point of our theory, i.e., it is considered that the inflow velocity at the periphery of the plume is a fraction $\alpha(\theta)$ of the maximum plume centreline velocity according to

$$\frac{dQ}{dx}=\int_0^{2\pi}\alpha(\theta)\ u_m\ b\ d\theta=\alpha_{\text{int}}(u_m\ b). \tag{7}$$

Then, using Boussinesq approximation, the equations of conservation of mass, x-momentum and buoyancy can be written as

$$\frac{d(\pi\ u_m\ b^2)}{dx}=\alpha_{\text{int}}(u_m\ b)$$

$$\frac{d(\pi\ u_m{}^2\ b^2)}{dx}=\frac{2\ g\ q_0\ H_0}{(H_0+H-x)\ \left(u_m(1+\lambda^2)^{-1}+u_b\right)}$$

$$\pi\Delta\rho_m\lambda^2b^2\left(\frac{u_m}{1+\lambda^2}+u_b\right)=\frac{\rho_a q_0 H_0}{(H_0+H-x)}$$

Now, we consider that the bending of the plume is primarily due to an inhibition of the entrainment coefficient in the part of the plume which is nearer to the other. We also consider that all transversal momentum entrained in the plume is retained by its elements as they rise. If M_c denotes the transversal entrained momentum, then

$$M_c = \int_0^x \int_0^{2\pi} \rho_a(\alpha(\theta)\ u_m b)\ \alpha(\theta)\ u_m \cos(\theta)\ d\theta\ dx \tag{11}$$

and the plume deflection angle can be approximated by $\tan(\phi) = M_c/M$.

Table 2
Plume deflection angle.

Case	i	j	K	l	m	n
ϕ_{exp}	2.9	3.0	1.1	2.3	1.1	1.7
ϕ_{theor}	2.5	3.2	2.3	2.4	2.3	2.3

In the present work, $\alpha(\theta)$ has been approximated by

$$\frac{\alpha_{\max} - \alpha_{\min}}{\pi}\theta + \alpha_{\min},\ 0 < \theta < \pi \tag{12}$$

$$= 0.06 - 1.15W_e,\ \alpha_{\max} = 0.065 \tag{13}$$

We have, thus, proposed a very simple linear correlation for the prediction of the entrainment coefficient behaviour.

The results are shown in Table 2.

It is clear from Table 2 that the results are good provided *We* number is not too small. In fact, cases *k* and *m* were the only ones where the model did no show to be appropriated. Under those conditions, *We* is really too small and difficult to incorporate in the theory.

CONCLUDING REMARKS

The present work has established a firm connection between the bending angle of a bubble plume and the values of *Fr* and *We* numbers. The work has been important in identifying some relevant parameters to the problem and in working out a strategy to determine values of the entrainment coefficient in future more sophisticated formulations of the problem. Also, a very simple theory based on integral methods has been advanced which provides good results.

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