SEGRE-SILBERBERG EFFECT AND GENERATION OF TURBULENCE IN SHEAR FLOW

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Interpretation is given of the Segre-Silberberg effect[1] (expressing itself in the specific radial migration of neutrally suspended spherical particles in the Poiseuille flow) in the light offered in 1980s model[2] of generation of turbulence in shear flow.

MODEL OF TURBULENCE GENERATION IN SHEAR FLOW

As it is a matter of common knowledge, based at the model of homogenous continuous flow of deformable area, theoretical hydrodynamics assigns laminar shear flow of viscous liquid to the category of vortex motion[3]. It is also known that rotational component of general motion of an infinitesimal element of fluid is equal to half of vorticity, although, within the framework of the model mentioned and corresponding mathematical formalism consequences of vorticity are reduced only to permanent shear strain of laminar flow (without actual rotation of fluid volumes of any scale). Besides, it is agreed that transition of such a virtual rotation to actual one is possible if a spherical element of the fluid is suddenly solidified[3].

The model[2] considers idealized laminar shear flow mentioned as a metastable medium, actual realization “vortex generation” potential thereof is possible only at presence of "nuclei" of a new "vortex" phase in the form of discrete non-deformable elements. As applied to a real liquid the consideration is offered of similar "phase conversion" through two possible levels of superposing on a homogeneous continuous flow of discrete structure with non-deformable elements: micro-level - connected with molecular structure of the liquid and macro-level - connected with presence of suspended micro-particles.

Here, concerning interpretation of the Segre-Silberberg effect, consideration of the last level is more suitable. More so that such a case deserves of general interest also as full prevention is practically impossible of the presence in usually considered as a pure real viscous flows of admixed micro-particles (of the scale of μm and less) such as corrosion products, micro-bubbles often behaving as a solid spheres, fine dust and others. Besides, the fact is also established experimentally[4] of rotation of firm spheres neutrally suspended in the Poiseuille flow just in line with the theory mentioned.

Rotation of suspended micro-particles leads to generation of vortex filaments along the rotation axis's and to superposition on shear flow of the system of ordered (identically oriented) vortex filaments. At the same time, according to general regularities of dynamics of such a systems[3] they tend to integrated rotation around so-called centroid of the vortexes. The model[2] attributes existence of the centroid of micro-vortexes to velocity profile of basic laminar flow assuming saturation of shear zone by uniform rotating liquid micro-elements. Within shear zone such a centroids form a surface which in connection with certain peculiarities (reflected below) may be named as an inversion surface.
So, according to the model mentioned, in the shear zone the micro-vortexes simultaneously with individual rotation are subjected to specific collective interactions. As a result the micro-vortexes suspended outside of the inversion surface tend to lead basic flow and, on the contrary, the micro-vortexes suspended between the inversion surface and the friction boundary tend to lag behind. Consequently, the sign changes of influence of the vortex system on individual micro-vortexes just at the inversion surface. A distance between the inversion and friction surfaces (which correspondingly may be named as an inversion thickness) is determined based at regularities of dynamics of ordered vortex systems[3] by the following equation[2]:

$$y_i = \frac{\int_o^{\delta} \omega y dy}{\int_0^{\delta} \omega dy} \quad (1)$$

where ω is angular velocity of the liquid micro-element, y is co-ordinate normal to the friction surface and δ is distance between the friction surface and the point of straightening of the velocity profile or reverse of the sign of the velocity gradient (in the case of the profiles without straightening or reverse of the sign of the velocity gradient δ is distance between boundaries of the flow, in the case of boundary layer flow δ is infinity).

For instance, in the Couette flow with linear velocity profile the angular velocity of the fluid micro-element is equal:

$$\omega = -\frac{1}{2}\frac{\partial u}{\partial y} = -\frac{u_h}{2h} \quad (2)$$

where h is distance between parallel friction surfaces, u is local velocity and u_h is velocity of the motive surface.

Correspondingly, the inversion thickness is equal:

$$\mathrm{y_i} = \frac{\int_0^{\mathrm{h}} \left(-\frac{\mathrm{u_h}}{2\mathrm{h}}\right)\mathrm{ydy}}{\int_0^{\mathrm{h}} \left(-\frac{\mathrm{u_h}}{2\mathrm{h}}\right)\mathrm{dy}} = \frac{\mathrm{h}}{2} \quad (3)$$

So, in the Couette flow the inversion surface is located midway between friction surfaces. Alongside, similar computations show that in the Poiseuille flow in the cylindrical channel the inversion surface is located at two thirds of the radius from the channel axis, in the Poiseuille flow between infinite parallel plates there are two inversion surfaces located at one sixth of the width from each of friction surfaces, in the downward laminar liquid film the inversion thickness is equal to one third of the film thickness. It is noteworthy also that in all cases inversion thickness is irrespective to the scale of the micro-vorticity.

In addition, it deserves of special interest that in the case of the boundary layer flow equation (1) reduces to equation for displacement thickness attaching new aspect to this basic parameter. At the same time, inversion thickness should be considered as more general parameter covering all types of laminar shear flow[2].

The model[2] considers the phenomena described above as a possible basis for development of turbulent transition in laminar shear flow. According to the model, permanent generation by shear flow with discrete non-deformable micro-elements of ordered micro-vorticity periodically may be "discharged" through certain mechanism (which still remains to be fully clarified[5]) into the first (in laminar flow) or any subsequent (in turbulent flow) large-scale vortex which is followed by a cascade of its splitting.

It is necessary to note also that the approach connecting turbulence with the critical phenomena during phase conversions independently was again put forward last years[6.] New model[7] also is offered connecting turbulence generation with transition of ordered vortex structures into vortex ensembles.

At the same time among the new investigations the results[8] deserves special attention presenting experimental and theoretical evidences of a strong analogy between the statistical behaviour of a confined turbulent flow and that of a model of critical behaviour of a ferromagnet. It was found that the power consumption measured in the turbulent flow experiments and the magnetization at the critical point of the ferromagnet have probability distributions of the same rather specific form, irrespective of Reynolds number on the one hand and system size on the other. The distributions both have non-gaussian tails that characterize the rare large-amplitude fluctuations. In this region, the scaled distributions for the two systems collapse onto a single universal curve over at least four orders of magnitude. This suggests a basic similarity in the finite-size corrections to the fluctuation statistics in the limit of infinite system size (for the magnetic system) or infinite Reynolds number (for turbulent flow).

A priori considering magnetic systems near the second-order phase transition and developed turbulence as microscopically very different physical systems similarity of the two functions mentioned is evaluated as surprising and the main conclusion is made about establishment of a new type of universality in the turbulence[8-9]. At the same time in the light of the model[2] there it exists the reason to consider just similarity between dynamics of formation of the rare large-scale turbulent vortexes and critical transition at magnetization of ferromagnet.

As it is known[10] the magnetization of ferromagnet by ordering of magnetic carriers under influence of the external magnetic field immediately is connected with corresponding ordering of micro-vorticity of the same micro-particles. Corresponding change of angular momentum at micro-level about the axis of the external field leads to occurrence of equal but opposite in sign compensating angular momentum imposed to the ferromagnet as a whole. Besides, if the ferromegnet is free to move the magnetization process results its integrated rotation (Einstein-de Haas effect).

So, dynamics of the magnetization of ferromagnet includes the transition of angular momentum from micro-level to macro-level and establishment of similarity of dynamics of this process with dynamics of formation of the large-scale turbulent vortexes may be interpreted as evidence of existence in the latter of certain role of the same type of transition. Correspondingly, in such a case the results[8] may be considered as supporting the model[2]. Unfortunately, most likely because of lack of information, the model[2] was not used by authors of the works mentioned through analysis of the problem and interpretation of the results received.

GENERATION OF ORDERED MICRO-VORTICITY AND SEGRE-SILBERBERG EFFECT

Now it is appropriate to trace in the framework of the model[2] behaviour of neutrally suspended macro-spheres (with diameters of order 1.0 mm and more) in laminar shear flow reflected through the Segre-Silberberg effect[1,4,11-12]. As firm macro-sphere neutrally suspended in the laminar shear flow rotates just in line with the theory[4] it should be subjected to influence of vortex system together with micro-vortexes. However, if the occurrence of the collective interactions may rather exactly correspond to regularities of the theory of ideal liquid[3] further motion of individual vortexes considerably depends from viscous forces.

As it is shown through corresponding analysis[5] in this connection effect of the field of collective interactions drastically depends from the size of the individual vortex. Besides, hindering role of viscosity is many times lower in the case of macro-sphere creating possibility of noticeable shifting of the latter relative to the basic flow (with almost full preservation of integrity of the micro-vortexes within the same flow). So, in the framework of the model[2] macro-sphere has exclusive possibility to shift relative to the basic laminar shear flow under influence of ordered micro-vorticity.

As a result rotating macro-spheres neutrally suspended outside of the inversion surface lead basic flow and, on the contrary, the macro-spheres lag behind suspended between the inversion surface and the friction surface. Opposite directions of the macro-spheres shifting relative to the basic flow in combination with identical directions of these rotation create lateral forces of opposite directions at different sides from inversion surface.

According to the most detailed experimental investigation[4] of the Segre-Silberberg effect in the Poiseuelle flow the macro-spheres leading the basic flow are subjected to lateral forces directed to the tube wall and others lagging behind the basic flow are subjected to lateral forces directed to the tube axis. So, direction of radial migration of the neutrally suspended macro-spheres in laminar shear flow always is targeted to the inversion surface. Finally, in accordance to this consequence of the model[2] in the Poiseuelle flow neutrally suspended micro-spheres are concentrating in narrow annular zone just at two thirds of the radius from the channel axis[4].

The results following from the model[2] are corroborated also by experimental data reflecting peculiarities of the Segre-Silberberg effect in the Couette flow[11]. In full accordance to equation (3) in the Couette flow neutrally suspended micro-spheres are concentrating at midway between friction surfaces.

Important results are presented also in investigation[12] devoted to experimental study of individual trajectories of neutrally suspended micro-spheres in the Poiseuelle flow. Firstly it should be mentioned experimental establishment of the absence of the Segre-Silberberg effect when rotation of the micro-sphere is prevented (achieved through drilling a fine hole in one side of the macro-sphere). The second qualitative result of the investigation is connected with establishment of existence of the same radial migration effect in the case of passage through the experimental cylindrical channel of single macro-sphere. This result is especially important just in the light of the model[2].

An agreement between the experimental data[1,4,11-12] and predicted by the model[2] behaviour of the macro-spheres obviously may be considered as corroboration of the theory. Correspondingly, it should be concluded also that the Segre-Silberberg effect unambiguously is connected with generation in the shear flow of ordered micro-vorticity. However, if this is the case manifestation of the Segre-Silberberg effect through passage of the single macro-sphere should be considered as an evidence of generation of ordered micro-vorticity in the basic laminar flow[11] itself (if in the case of passage of large number of the macro-spheres generation of ordered vorticity may be attributed to the rotating macro-spheres themselves).

As a whole presented interpretation by the model[2] of the Segre-Silberberg effect may be considered as an evidence of existence of the preparatory stage of formation of integral large-scale vortex even in a laminar viscous flow with rather small Reynolds numbers though this process is very slow in similar conditions.

CONCLUSION

Unambiguous interpretation of the Segre-Silberberg effect through prediction of concentration of neutrally suspended spherical particles at the inversion surface of the laminar shear flow and establishment of similarity between dynamics of formation of the large-scale turbulent vortexes and critical transition at magnetization of ferromagnet may be considered as supporting evidences for the turbulent transition model connecting this process to generation in the shear flow and further development of ordered micro-vorticity. It is appropriate to perform new subsequent theoretical and experimental studies by the goal of further verification and development of the approach considered.

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