TWO-PHASE MODEL OF FILTRATION OF HEAVY LIQUID CONTAMINATION IN LAYERED AQUIFER

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ABSTRACT. The filtration process of heavy liquid contamination in the layered sloping freshwater reservoir at presence of infiltration through its roof is studied on base of mathematical and computer simulation. The heavy contamination is or a highly concentrated salty solution (the salty water) or an organic liquid. The contamination penetrates into reservoir or through the part of its roof or comes into some layers from their inputs with the side flow. Such process of gravitational redistribution of two liquids with various densities is being characterized by instability of flow and, irrespective of type of contamination, can be described within the two-phase filtration theory when phases are presented as non-intermixing liquids. The fundamental properties of the problem solution, which are induced by the layered heterogeneity of reservoir, were studied. In particular, it was shown that the generation of movable reflected jumps of saturation could take place. Effective numerical models and algorithms are developed with use of a priori information about the solution behavior. The influence of a reservoir structure on distribution of heavy contamination was investigated by means of computational experiments. Some examples of numerical investigations are presented.

1. MATHEMATICAL THEORY

Let's consider the aquifer with the layered structure. Its layers are hydraulically interconnected and can contain inhomogeneous inclusions (with respect to absolute permeability). At the initial stage of filtration the heavy contamination penetrates into a water phase because of difference of their densities. It leads to formation of the unstable jet flow, in which the moving phases have opposite directions. Such process can be described by the equations of two-phase filtration with relative phases permeability of contamination k_1^* and water k_2^*, which are depended linearly on saturation of appropriate phases [Barenblatt *et al.*, 1984]. Hereinafter a fraction of the elementary macroscopic porous volume, occupied by contamination, will be named as the saturation and designated as S.

In case of the organic pollution there is some limit value S^* of saturation (dependent on structure of the porous medium) when the connectivity of water phase will be infringed, and the water loses its mobility because of capillary forces, i.e. $k_2^* = 0$ for $S \geq S^*$. At the next stage, when the water is compelled to go inside the channels, occupied by organic liquid, the action of capillary forces becomes more intensive. Hereupon the dependencies of the phase permeability become nonlinear and hydrodynamic mobility of contamination ends at significance of the irreducible saturation S_*. Thus, on the forward front of the organic contamination field the phases permeability are linear, and on the back front they are nonlinear in respect to saturation S. In case of the salty water the distribution of phases in pores is not depended on capillary forces and each phase saves mobility at any saturation. Their phases permeability remain linear both on the forward front and on the back one. Equations describing process of two-phase filtration within large-scale approximation can be written as:

$$\mathrm{div}\vec{\mathbf{V}} = 0, \qquad \mathrm{div}\vec{\mathbf{V}}_1 + m\frac{\partial S}{\partial t} = 0 \tag{1}$$

Here $\vec{\mathbf{V}} = -k^* k / \mu_2 \,(\mathrm{grad} P + \rho\, \vec{\mathbf{g}})$, $\vec{\mathbf{V}}_1 = f \cdot \vec{\mathbf{V}} + k\rho_{1-2}\, \Psi \cdot \vec{\mathbf{g}}$, $\vec{\mathbf{V}} = \vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2$, $\vec{\mathbf{V}}$ - a total filtration velocity, μ_i, ρ_i, $\vec{\mathbf{V}}_\mathbf{i}$ - i-phase viscosity, density and velocity, m - porosity; P - pressure, k - absolute permeability, $\rho = f \rho_1 + (1-f)\rho_2$ - density of two-phase mixture, $k^* = \mu \cdot k_1^* + k_2^*$ - relative mobility, $\vec{\mathbf{g}}$ - acceleration of gravity, $\Psi = k_2^* f$, $\mu = \mu_2/\mu_1$, $\rho_{1-2} = \rho_1 - \rho_2$, $f = \mu \cdot k_1^* / k^*$ - the Buckley-Leverett function.

Now we'd like to note **some properties** of desired solution. It can be easily show that the function Ψ tends to zero on extremities of an interval $(0, S^*)$. Because of this property the above-mentioned nature of the jet flow takes place when there is a movement of phases in opposite directions. So, firstly, we shall consider S - dependence of the filtration velocity $\vec{\mathbf{V}}_1$. Fig. 1(a) presents schematically the qualitative graphs of its Z-component V_{1z}, where OZ is axes of coordinates orthogonal to the reservoir roof. Various significance of the total flow V_z are corresponded to four possible situations. Here solid curves and dotted ones designates the cases when the motion of both phases occur in identical and opposite directions accordingly. The direction of contamination flow and the total flow is identical (curve 2) and opposite (curve 3). In both these cases the function V_{1z} attains its maximum $\tilde{\mathrm{V}}_{1z}$ at some significance $\bar{S} < S^*$. We note also, that S^* is equal to 1 for the salty water.

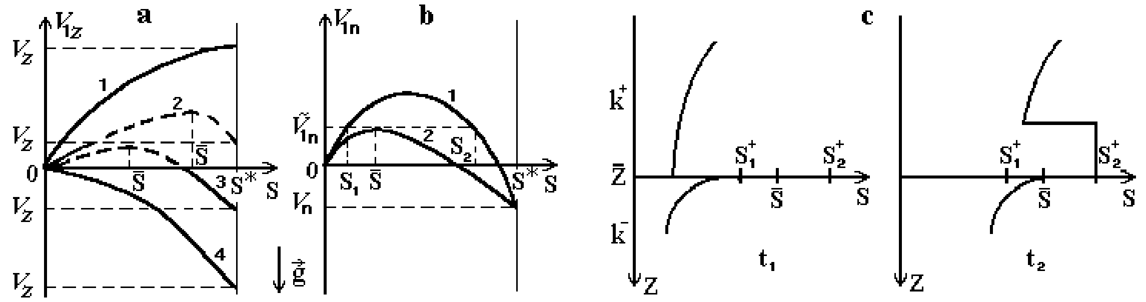


Figure 1. To properties of the system solution. **a:** S-dependence of filtration velocity V_{1z} at various values of V_z; **b:** $\mathrm{V}_{1n}(S)$ at the line of discontinuity k ; **c:** profiles of saturation S under generation of the reflected jump at line of discontinuity k ($k^+ >> k^-$).

Further, in considered problems the absolute permeability k belongs to a class of sectionally continuous functions. Therefore, the equalities of pressures $P^+ = P^-$ and of normal components of the liquid flows $V_n^+ = V_n^-$, $V_{1,n}^+ = V_{1,n}^-$ should be satisfied at line of discontinuity k. In Fig. 1(b) the characteristic graphs of S- dependencies V_{1n}^- and V_{1n}^+ at this line are schematically shown. Here the curve 1 corresponds to the contamination flow at the side of area of greater permeability. It is easy to see in figure, that at the line of discontinuity the function S loses its continuity, that the saturation at such side of area can not belong to the interval (S_1, S_2). The latter leads to generation of a movable jump $S_j = S_2$, which is moving up in the layer of high permeability, see Fig. 1(c). Here saturation profiles before and after formation of the jump correspond to times t_1 and t_2.

The presence of such jump leads to non-standard boundary conditions on a region Γ of the reservoir roof through which the contamination comes into it. Really, on this boundary it is impossible to set arbitrary an intensity of the pollution source. Firstly, it is due to evident condition that this intensity can not exceed magnitudes $\tilde{\mathrm{V}}_{1z}$. Secondly, the reflected movable jump S_j, reaching the boundary Γ and defined from solution of equations (1), decrease of the source intensity. In detail the statement of such boundary conditions is discussed in [Chekalin & Konyukhov, 1999].

2. SOME RESULTS OF COMPUTER SIMULATION. CONCLUSIONS

Equations (1) are solved by the finite-difference methods. The numerical models are developed with use of a priori information about properties of the system solution. Such approach gave possibility to obtain the highly effective algorithms and to create the computer programs for calculations of the migration process of the contamination field. Using these programs the series of computational experiments was done, some results of which are presented in Figs. 2-4.

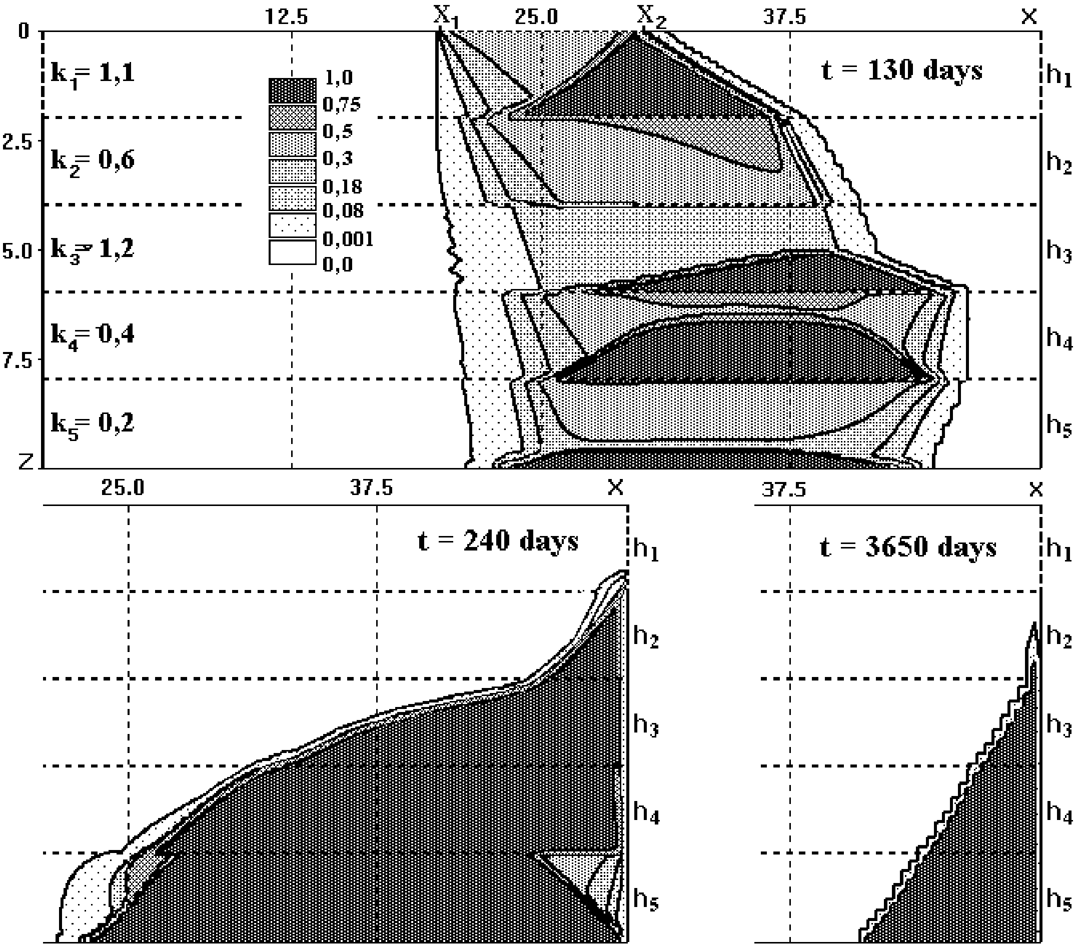


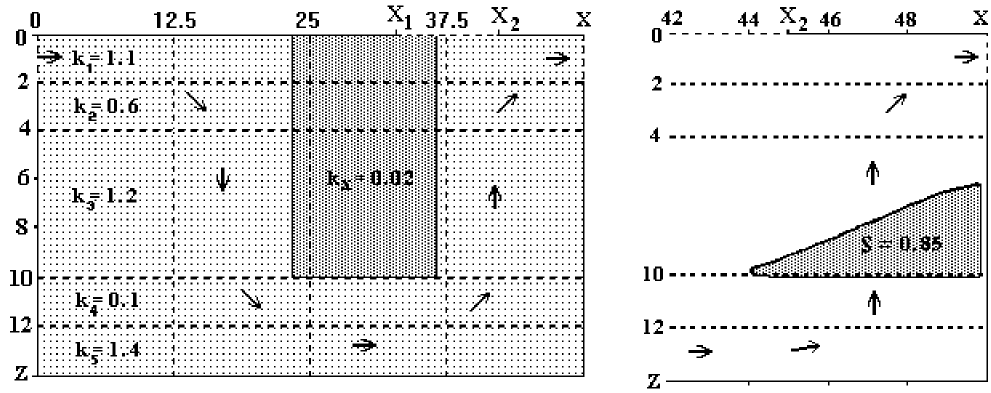
Figure 2. Migration of the contamination field in horizontal reservoir at presence of side flow, when the first upper layer is penetrable only and **the salty water** comes into reservoir through the part X_1X_2 of its roof.

Here the penetrable areas on the sides of reservoir and also the boundaries between layers are shown as dotted lines. To facilitate observations the areas of the saturation S located between adjacent isolines are filled. Patterns of shading and the ranges of S are given in figures. Coefficients of absolute permeability k_i (m day^{-1}) of layers are printed there. **Some resume** from an analysis of computational experiments can be formulated as follows.

• Filtration process of heavy contamination has “fast” and “slow” stages. At “fast” (initial) stage the large mass of contamination comes through the highly permeable zones of reservoir and accumulates at its bottom. At the “slow” stage the secondary (residual) pollution of layers occurs owing to effluence of contamination from the low-permeable zones and movement of the contamination field because of the side flow and the reservoir declination (see Fig. 2).

• The presence of impenetrable vertical barriers on the path of the moving contamination field leads to accumulation of the contamination mass near their boundaries (Fig. 2). This field under certain conditions can remain practically unmovable. In Fig. 2 at moments 240 and 3650 days are shown only those areas of reservoir, where the contamination field is located. Rising saturation jumps reflected from impenetrable bottom and from boundaries of discontinuity of absolute permeability are well visible there (at t = 130 days).

• The next interesting result is that in a vertical flow of fresh water the contamination field can lose mobility in the high-penetrable layer adjoining to the low-penetrable one (see Fig. 3). The direction of flow formed by the low-penetrable inclusion is schematically shown in this figure by arrows.



Scheme of reservoir structure and velocity field

Figure 3. Formation of **the unmovable salty water field** in with high permeable layer at presence of the poorly penetrable zone.

• The distribution processes of organic and non-organic contamination are qualitatively identical. The quantitative distinction between these processes is, firstly, that the migration of organic contamination is much more long-term. Secondly, moving through reservoir an organic contamination field generates the residual pollution track due to capillary effects (see in Fig. 4).

• The structure exerts essential influence on distribution process of the contamination field. To investigate this fact the reservoir containing the low-penetrable inclusions («porous blocks») was considered. Especially we was interested to study the motion of heavy contamination in «fissure-porous» reservoir under and without side flow, at presence and at absence of infiltration through its roof. By means of computational experiments it was shown that the side flow shifts the contamination field, but its influence is insignificant in increasing of the contamination effluent flow from blocks. Very small intensity of the contamination flow from blocks leads to rather long-term secondary pollution of groundwater layers at the “slow” stage of filtration process. Our investigation have shown also that the infiltration forces the displacement of contamination from blocks and layers.

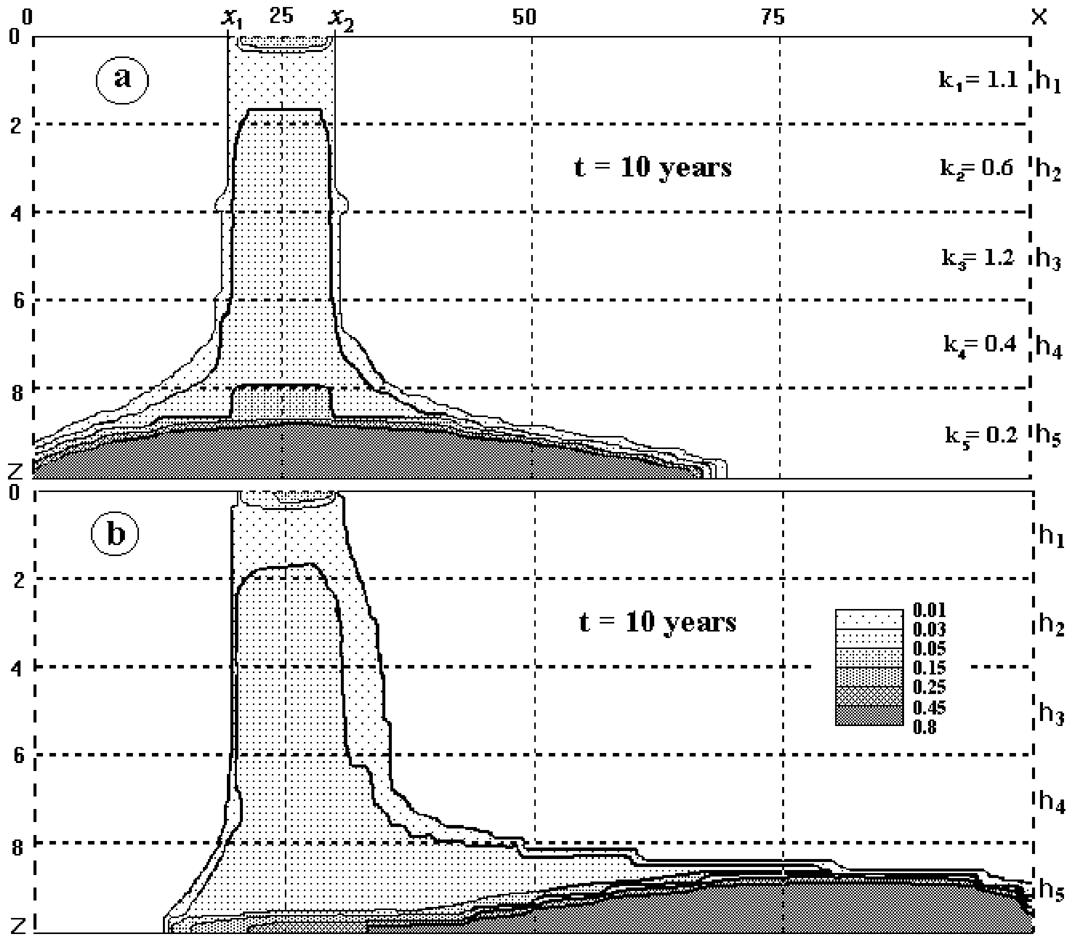


Figure 4. Influence of declination on motion of **the organic contamination**, penetrating into reservoir through part X_1X_2 of its roof at absence of side flow:
a: $\alpha = 0$ - the horizontal reservoir, **b:** $\alpha = 0.2$ - the sloping reservoir.

3. ACKNOWLEDGEMENT

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