NATURAL CONVECTION IN A SHALLOW HORIZONTAL RECTANGULAR CAVITY HEATED FROM A VERTICAL SIDE

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INTRODUCTION

If current search testifies to an increasing rise in the number of studies related to the porous layers saturated with a non-Newtonian fluid, it is not the case for the non-Newtonian fluid-filled enclosures where the lack of studies is serious. For instance, the understanding of the non-Newtonian fluid flows is of practical interest in paper making, drilling of petroleum products, slurry transporting, and processing of food and polymer solutions. The main objective of this paper is to address the question of implications of the non-Newtonian behavior effects on the natural convection heat transfer within an horizontal rectangular enclosure submitted to a constant heat flux from the left vertical side while its horizontal walls are insulated (fig. 1).

GEOMETRY AND GOVERNING EQUATIONS

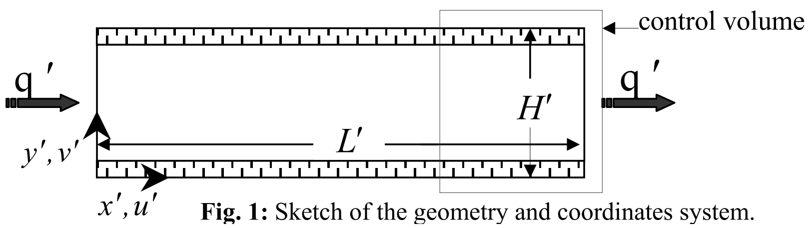

Fig. 1: Sketch of the geometry and coordinates system.

Under the assumptions commonly used in natural convection, the dimensionless governing equations written in terms of vorticity (Ω), temperature (T) and stream function (ψ) are:

$$\frac{\partial \Omega}{\partial t}+\frac{\partial u\Omega}{\partial x}+\frac{\partial v\Omega}{\partial y}=Pr\left[\mu_a\left[\frac{\partial^2\Omega}{\partial x^2}+\frac{\partial^2\Omega}{\partial y^2}\right]+2\left[\frac{\partial\mu_a}{\partial x}\frac{\partial\Omega}{\partial x}+\frac{\partial\mu_a}{\partial y}\frac{\partial\Omega}{\partial y}\right]\right]+S_\Omega \tag{1}$$

$$\frac{\partial T}{\partial t}+\frac{\partial uT}{\partial x}+\frac{\partial vT}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-\Omega \text{ with} \tag{3}$$

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x},\ \Omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y},\ \mu_a=\left[2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right]^2\right]^{\frac{n_0-1}{2}} \text{ and} \tag{4}$$

$$S_\Omega=Pr\left[\left[\frac{\partial^2\mu_a}{\partial x^2}-\frac{\partial^2\mu_a}{\partial y^2}\right]\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right]-2\frac{\partial^2\mu_a}{\partial x\partial y}\left[\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right]\right]+Pr\,Ra\frac{\partial T}{\partial x} \tag{5}$$

The non-dimensional appropriate boundary conditions are used:

$$u=v=\Psi=\partial T/\partial x+1=0 \ \ for\ x=0\ and\ A\ ;\ u=v=\Psi=\partial T/\partial y=0\ \ for\ y=0\ and\ 1 \tag{6}$$

The parameters $A=L'/H'$, $Pr=\frac{(k_0/\rho_0)H'^{2-2n_0}}{\alpha^{2-n_0}}$ and $Ra=\frac{g\,\beta\,H'^{2n_0+2}q'}{(k_0/\rho_0)\alpha^{n_0}\lambda}$ represent respectively the cavity aspect ratio and the generalized Prandtl and Rayleigh numbers.

NUMERICAL SOLUTION

The governing equations are solved numerically by using the well known second order central finite difference method for a regular mesh size. The integration of the equations (1) and (2) is performed with the Alternating-Direction Implicit method (***ADI***). The equation (3) is treated by the Point Successive Over-Relaxation method (***PSOR***) with an optimum relaxation factor calculated by the Franckel formula[1]. The uniform grids of 241×41 are sufficient to model accurately the flow and temperature fields in the cavity with $A = 8$

PARALLEL FLOW ANALYSIS

In the case of a shallow cavity (A>>1), the flow presents a parallel aspect and a thermal stratification[2], which leads to the flowing simplifications:

$$u(x,y)\approx u(y),\ v(x,y)\approx 0 \text{ and } T(x,y)=C\,x+\theta(y) \tag{7}$$

where the constant C is obtained by integrating the energy equation on the control volume of fig.1

$$C=\int_0^1 u(y)\,\theta(y)\,dy-1 \tag{8}$$

The new resulting non-dimensional governing equations are:

$$\frac{\partial^2}{\partial y^2}\left[\left|\frac{\partial u}{\partial y}\right|^{n_0-1}\frac{\partial u}{\partial y}\right]=CRa \text{ and } Cu=\frac{\partial^2\theta}{\partial y^2} \tag{9}$$

with

$$u=\partial\theta/\partial \mathrm{y}=0 \text{ for } y=0 \text{ and } 1 \text{ (as boundary conditions)} \tag{10}$$

and

$$\int_0^1 u(y)\,dy=0 \text{ (as return flow condition)} \tag{11}$$

The non-linearity of the fluid behavior and the change of the velocity gradient sign due to the return flow, imposes that the velocity expressions are different depending on whether $0\le y\le y_0$, $y_0\le y\le y_1$ or $y_1\le y\le 1$, where y_0 and $y_1=1-y_0$, are the values of the transverse co-ordinate correspond to the velocity extremum positions. They are derived from Eq. (11) which is solved by using a combination of the Regula-Falsi and Wegstein iteration methods and the Gauss-Legendre integration method. To simplify the velocity and temperature expressions, it is useful to introduce the functions $f(y)=y^2/2-y/2+y_0y_1/2$. Thus for $0\le y\le y_0$, we obtain

$$u(y)=C^{1/n0}\ Ra^{1/n0}\left[\int_0^y[f(y)]^{1/n_0}\,dy\right] \tag{12}$$

$$\theta(y)=C^{1+1/n0}\ Ra^{1/n0}\left[\int_0^y\left[\int_0^y\left[\int_0^y[f(y)]^{1/n_0}\,dy\right]dy\right]dy\right]+\theta(0) \tag{13}$$

for $y_0\le y\le y_1$

$$u(y)=C^{1/n0}\ Ra^{1/n0}\left[\int_0^{y_0}[f(y)]^{1/n_0}\,dy+\int_y^{y_0}[-f(y)]^{1/n_0}\,dy\right] \tag{14}$$

$$\theta(y)=C^{1+1/n0}\ Ra^{1/n0}\left[\frac{(y-y_0)^2}{2}\int_0^{y_0}[f(y)]^{1/n_0}\,dy+\int_{y_0}^{y}\left[\int_{y_0}^{y}\left[\int_y^{y_0}[-f(y)]^{1/n_0}\,dy\right]dy\right]dy\right.$$
$$\left.+(y-y_0)\int_0^{y_0}\left[\int_0^y[f(y)]^{1/n_0}\,dy\right]dy+\int_0^{y_0}\left[\int_0^y\left[\int_0^y[f(y)]^{1/n_0}\,dy\right]dy\right]dy\right]+\theta(0) \tag{15}$$

and for $y_1\le y\le 1$

$$u(y)=C^{1/n0}\ Ra^{1/n0}\left[\int_0^{y_0}[f(y)]^{1/n_0}\,dy+\int_{y_1}^{y_0}[-f(y)]^{1/n_0}\,dy+\int_{y_1}^{y}[f(y)]^{1/n_0}\,dy\right] \tag{16}$$

$$\theta(y)=C^{1+1/n0}\ Ra^{1/n0}\left[\frac{1}{2}(y-y_0)^2\int_0^{y_0}[f(y)]^{1/n_0}\,dy+\frac{1}{2}(y-y_1)^2\int_{y_1}^{y_0}[-f(y)]^{1/n_0}\,dy\right.$$
$$+\int_{y_1}^{y}\left[\int_1^y\left[\int_{y_1}^{y}[f(y)]^{1/n_0}\,dy\right]dy\right]dy+\int_{y_0}^{y_1}\left[\int_{y_0}^{y}\left[\int_y^{y_0}[-f(y)]^{1/n_0}\,dy\right]dy\right]dy$$

$$+(y-y_0)\int_0^{y_0}\left[\int_0^{y}[f(y)]^{1/n_0}\,dy\right]dy+(y-y_1)\int_{y_0}^{y_1}\left[\int_y^{y_0}[-f(y)]^{1/n_0}\,dy\right]dy$$

$$+\int_0^{y_0}\left[\int_0^{y}\left[\int_0^{y}[f(y)]^{1/n_0}\,dy\right]dy\right]dy+\theta(0) \qquad (17)$$

where

$$\theta(0)=-C^{1+1/n0}\,Ra^{1/n0}\left[\frac{(1/2-y_0)^2}{2}\int_0^{y_0}[f(y)]^{1/n_0}\,dy+\int_{y_0}^{1/2}\left[\int_{y_0}^{y}\left[\int_y^{y_0}[-f(y)]^{1/n_0}\,dy\right]dy\right]dy\right.$$

$$\left.+(1/2-y_0)\int_0^{y_0}\left[\int_0^{y}[f(y)]^{1/n_0}\,dy\right]dy+\int_0^{y_0}\left[\int_0^{y}\left[\int_0^{y}[f(y)]^{1/n_0}\,dy\right]dy\right]dy\right]-CA/2 \qquad (18)$$

RESULTS AND DISCUSSION

Let us mentioned, at first, that this study is conducted without considering the Prandtl number since for it's high values, which characterize the non-Newtonian fluids, the convection heat transfer becomes insensitive to any change in this parameter since the contribution of the convective terms of Eq. (1) becomes negligible. Thus, the control parameters of the study are the Rayleigh number Ra and the behavior index n_0. Compared to the case where the cavity is heated from the bottom, the convection takes place with the least small value of Ra (fig. 3). As it can be seen from fig. 2, the flow presents a parallel aspect and a thermal stratification in a large part of the cavity. This proves the existence of an analytical solution which is in good agreement with the numerical one (fig. 3). On the other hand the same figure shows that a decrease of n_0 enhances the convection heat transfer.

CONCLUSION

The study of the natural convection of non-Newtonian fluids confined in a horizontal shallow rectangular cavity heated from the left vertical side, is investigated numerically and analytically. It should be noted that the double objective of the study, which consists to prove the existence of an analytical solution for this problem and also the validation of the numerical code, is reached. In addition, the effect of the non-Newtonian behavior is such as the convection heat transfer is favored by the shear-thinning behavior ($0 < n_0 < 1$).

REFERENCES

1. Roache, P. J., Computational fluid dynamics, *Hermosa Publishers*, Albuquerque, New Mexico, 1982.
2. Bejan, A., Convection heat transfer, *John & Sons*, 1984.

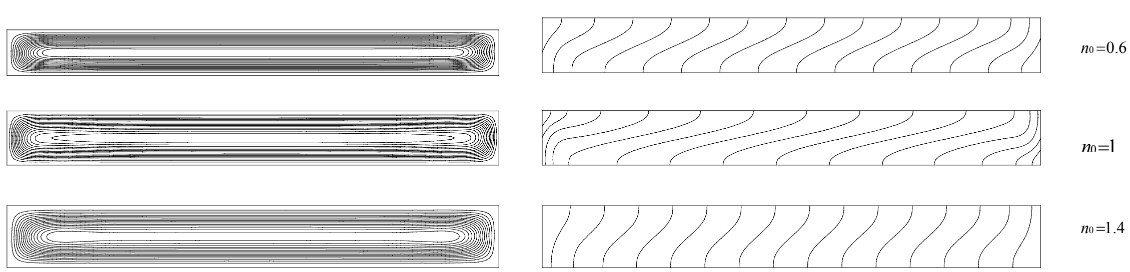

Fig.2: Streamlines (left) and isotherms (right) for Ra = 2000 and different values of n_0.

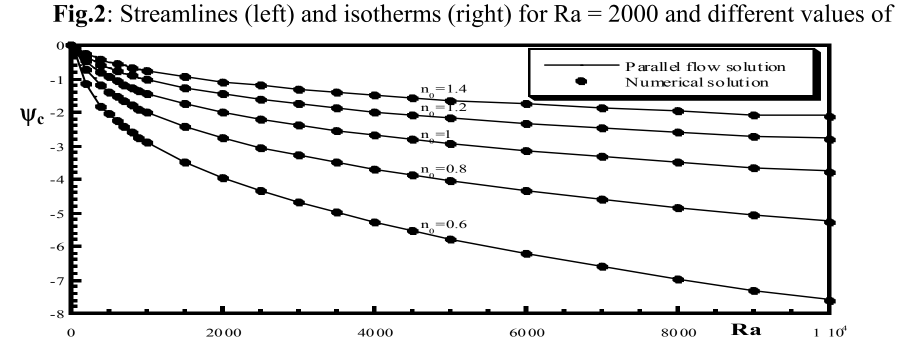

Fig. 3: Evolution of the stream function at the center of the cavity with the Raleigh number for various values of n_0.