Application of the modified method of discrete sources for solving the problem of wave scattering by group of bodies.

A.G.Kyurkchan, S.A.Manenkov

Moscow Technical University of Communications and Informatics,
Aviamotornaya, 8A, 111024, Moscow, Russia
tel:+7-495 -2737509, e-mail: kyurkchan@yandex.ru

Abstract

The new method for solving the problem of wave diffraction on a group of bodies of revolution is presented. The method is based on the simple algorithm and it allows to calculate the electromagnetic field and the pattern with high accuracy.

1 Introduction

The modified method of discrete sources (MMDS), offered in the paper [1], has been subsequently applied for solving a wide class of problems of diffraction theory, and in all the cases high efficiency of the method [2] has been shown. The uniform way of construction of the carrier of discrete (auxiliary) sources by means of analytical deformation of border of a scatterer is the main idea of the method. Thus a priori information of properties of analytical continuation of diffraction field inside the scatterer is materially used.

The major practical issue is how to apply MMDS for solving the problem of wave diffraction on a closely located group of bodies. In consequence of diffraction interaction of the bodies the picture of arrangement of the singularities of the analytical continuation of wave field inside each scatterer can significantly differ from that which takes place in the case of a single body. In the case of close location of the scatterers singular points start "to be multiply", i.e. the singularities inside one body generate the singularities inside the other. In the paper some modification of MMDS is realized. This makes the method efficient for solving the problem of wave diffraction on a closely located group of bodies. The essence of this modification is that the carrier of discrete sources for each body is constructed using usual scheme of MMDS. However the sources surrounding the singular points, which appear because of the interaction between the scatterers, are appended in addition to the basic sources. In the paper the effective numerical algorithm for finding the singularities based on the continuation by a parameter is offered.

2 The statement of the problem and main relations.

Let the group of two bodies of revolution is located on one axis and bounded by surfaces S_1 and S_2. We choose the system of coordinates so that the axis z coincides with the axis of revolution of the bodies. Assume, that the impedance boundary condition on the surfaces of the scatterers is satisfied:

$$\vec{n}_p \times \vec{E} = Z_p \vec{n}_p \times (\vec{n}_p \times \vec{H}), \quad p = 1,2 \, , \tag{1}$$

where Z_p is the impedance on the surface S_p and $\vec{n}_p$ is the outward normal. The secondary field, everywhere outside the domains of the bodies, obeys the homogeneous Maxwell equations and the attenuation condition at infinity.

Let's introduce the local systems of coordinates connected with each of the scatterer. We choose the origins of the systems inside the surfaces S_1 and S_2. Then the secondary field is equal to the sum of the fields scattered by each body:

$$\vec{E}^1 = \frac{\eta}{i}\nabla_p \times \nabla_p \times \sum_{p=1}^{2} \int_{\Sigma_p} \vec{J}_p G_p d\sigma \,, \tag{2}$$

$$\vec{H}^1 = k \cdot \nabla_p \times \sum_{p=1}^{2} \int_{\Sigma_p} \vec{J}_p G_p d\sigma \,, \tag{3}$$

where

$$G_p = \frac{\exp(-ikR_p)}{4\pi kR_p}, \quad R_p = \left|\vec{r}_p - \vec{r}\,'_p\right|, \quad \vec{r}\,'_p \in \Sigma_p, \quad p = 1,2. \tag{4}$$

Here Σ_p is the auxiliary surface of revolution located inside the initial surface S_p of the *p*-th body, $\vec{J}_p$ is the unknown current distributed on the surface Σ_p, k is the wave-number, η is the wave impedance of the medium. The expressions for the electric and magnetic field components in the spherical coordinate system connected with the given body, are presented in [2]. Similarly to the paper [2] we pass to the parametrical representation of the surfaces of the scatterers. Then for the surfaces S_1 and S_2 we get:

$$x_p = r_p \sin\theta_p \cos\varphi, \quad y_p = r_p \sin\theta_p \sin\varphi, \quad z_p = r_p \cos\theta_p, \quad p = 1,2\,, \tag{5}$$

where $r_p = r_p(\theta_p)$ are the equations of these surfaces in the local spherical coordinates. The auxiliary surface Σ_p has the following equations:

$$x'_p = \rho_p \sin\alpha_p \cos\varphi, \quad y'_p = \rho_p \sin\alpha_p \sin\varphi, \quad z'_p = \rho_p \cos\alpha_p, \tag{6}$$

where

$$\alpha_p = \arg\xi_p(t_p), \quad \rho_p = \left|\xi_p(t_p)\right|, \quad \xi_p(t_p) = r_p(t_p + i\delta_p)\exp(it_p - \delta_p), \quad p = 1,2. \tag{7}$$

In the formulas (7) δ_p is the positive parameter responsible for the degree of deformation of the contour of the *p*-th body cross-section, $t_{1,2} \in [0,\pi]$. The choise of the parameters δ_p is described in [2]. By analogy with the paper [2] we present the unknown currents on the surfaces $\Sigma_{1,2}$ in the form:

$$\vec{J}_p = \vec{I}_p \big/ (\chi_p \rho_p \sin\alpha_p), \quad \chi_p = \sqrt{\rho_p^2(\alpha_p) + \rho'(\alpha_p)^2}\,, \tag{8}$$

where strokes mean the derivatives with respect to the corresponding arguments and

$$\vec{I}_p = I_{p1}\frac{\rho'_p(\alpha_p)}{\rho_p(\alpha_p)}\vec{i}_{\rho_p} + I_{p1}\vec{i}_{\alpha_p} + I_{p2}\vec{i}_\beta, \quad p = 1,2. \tag{9}$$

From the formulas (2) - (9) it is easy to get the system of integral equations relative to the Fourier harmonics of four unknown currents $I_{11}^m, I_{12}^m, I_{21}^m, I_{22}^m$. In the matrix form the system looks like:

$$\mathbf{KI} = \mathbf{B}\,, \tag{10}$$

Where the matrix consists of four blocks:

$$\mathbf{K} = \begin{pmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{pmatrix}, \tag{11}$$

where each block is:

$$\mathbf{K}_{pq} = \begin{pmatrix} \int_0^\pi K_{11}^{pq}(\theta_p, t_q) I_{q1}^m(t_q) dt_q & \int_0^\pi K_{12}^{pq}(\theta_p, t_q) I_{q2}^m(t_q) dt_q \\ \int_0^\pi K_{21}^{pq}(\theta_p, t_q) I_{q1}^m(t_q) dt_q & \int_0^\pi K_{22}^{pq}(\theta_p, t_q) I_{q2}^m(t_q) dt_q \end{pmatrix}, \quad p, q = 1, 2, \quad m = 0, \pm 1, \pm 2... \qquad (12)$$

The kernels K_{ij}^{pq} of the equations are similar to those presented in [2].

3 Numerical algorithm and some results.

The usual scheme of the numerical solution of the system (10) is described in [2]. Let's consider the problem of finding the singularities of the scattered field under diffraction on the closely located group of bodies. Assume that the body of revolution with the smooth border is placed near the other body, which is sharp-pointed. Thus, it is supposed, that the second body singularity is situated close to the surface of the smooth one. We name this singular point as the singularity-source. Suppose, that given singular point has polar coordinates (r_0, θ_0) in the coordinate system connected with the smooth body. As mentioned above, the singularity-source generates the additional singular point (singularity-image) inside the smooth scatterer, which would be absent under diffraction on this single body. To find the coordinates of the singularity-image we use the method of continuation by a parameter. As this parameter the distance r_0 is used. The coordinates of the singularity-source are also found numerically. The equation defining the coordinates of the singularity-image inside the smooth body has the following form:

$$f(\theta) e^{-i\theta} = r_0 e^{-i\theta_0}, \qquad (13)$$

where $f(\theta)$ is the equation of the contour of the smooth body cross-section in the polar coordinate system connected with given body. The equation (13) defines the value of the complex angle θ corresponding to the singularity-image. The polar coordinates of this point are accordingly equal to:

$$r_i = |\xi(\theta)|, \quad \theta_i = \arg \xi(\theta), \quad \xi(\theta) = f(\theta) e^{i\theta}. \qquad (14)$$

For further solution of the problem we surround all the singularities-images with the circles of the small radius. When these circles rotate they represent the toroidal surfaces with round sections. Thus the integrals over these toroidal surfaces are added to the expressions (2) and (3) for the scattered field. This gives rise to the additional discrete sources in the presentation for the secondary field.

Comparison of the stated algorithm with the pattern equations method (PEM) presented in the paper [3] has been performed. As an example we have considered the diffraction of the plane wave

$$\vec{E}^0 = \vec{i}_x \exp(-ikz), \quad \vec{H}^0 = \frac{1}{\eta} \vec{i}_y \exp(-ikz) \qquad (15)$$

on two identical superellipsoids. The equation of the contour of the superellipsoid cross-section is:

$$\left(\frac{x}{a}\right)^{2s} + \left(\frac{z}{c}\right)^{2s} = 1 \qquad (16)$$

The sizes of the bodies are $ka = 2.5, \quad kc = 5$ and $s = 10$. The minimal distance between the surfaces of the bodies is $kd = 0.02$. In Fig. 1 the angular dependence of the module of the pattern for the concerning group of bodies (solid curve) is presented. The dashed curve demonstrates the results obtained by means of PEM. It follows from the figure that the results of calculations coincide with high accuracy.

Fig. 2 illustrates the advantages of the modified MMDS in comparison with the usual algorithm, which does not consider the singularities-images inside the smooth body generated by the singularities of the other body with rough border. The figure shows the residual of the boundary condition on the contour of the cross-section of the spheroid near to which the double-cone is located. The double-cone is modeled by the generalized superellipsoid of the following kind:

$$\left(\frac{z-\nu x}{c}\right)^{2s}+\left(\frac{z+\nu x}{c}\right)^{2s}=1. \quad (17)$$

Axial incidence of the plane wave (15) is considered. The sizes of the bodies have the following values: semi axes of the spheroid are $ka=4$, $kc=2$, the maximal sizes of the double-cone along the coordinate axes are equal to 4, the parameter $\nu=1$ and $s=10$. The distance between the surfaces of the bodies is $kd=0.1$. Curve 1 in the figure corresponds to the standard MMDS, and curve 2 does to the modified algorithm considering the singularity-image on the small axis of the spheroid. The number of the additional sources surrounding the singularity-image is equal to 7. Note, that the full number of discrete sources in both cases are identical and equal to 267. It follows from the figure that the level of the residual obtained by the modified MMDS much less the level of the residual obtained with the use of the standard MMDS.

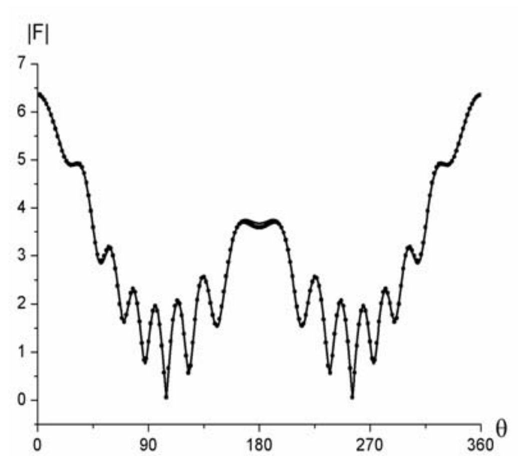

Fig.1

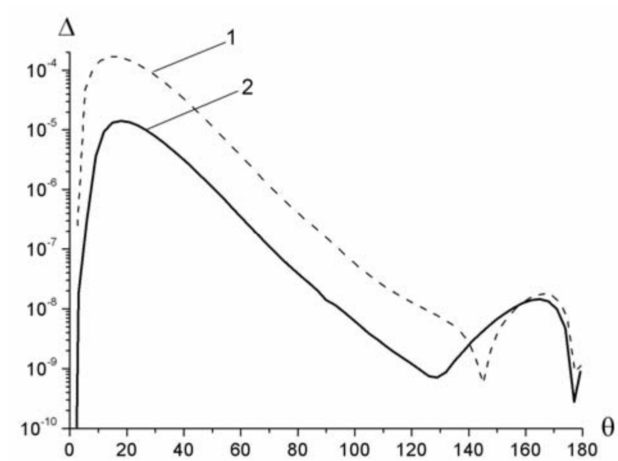

Fig.2

Acknowledgments

This work was supported by the Russian Foundation for Basic Researches (grants 06-02-16483 and 05-02-16931).

References

[1] A.G.Kyurkchan, S.A.Minaev, A.L.Soloveychik. “A modification of the method of discrete sources based on prior information about the singularities of the diffracted field”, Radiotekh. Electron., **46**, 666-672 (2001).

[2] A.P.Anyutin, A.G.Kyurkchan, S.A.Manenkov, S.A.Minaev. “About 3D solution of diffraction problems by MMDS”, JQSRT, **100**, 26-40 (2006).

[3] A.G.Kyurkchan, E.A.Skorodumova, “Modeling the characteristics of the waves scattering by a group of scatterers”, JQSRT, **100**, 207-219 (2006).