Simplex inversion of asteroid photometric lightcurves

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Abstract

We develop simplex inversion methods for asteroid photometric lightcurves in the case of limited and/or sparsely distributed observations. We show that the methods can be utilized in the computation of asteroid spins and convex shapes described using a finite number of triangles.

1 Introduction

In asteroid lightcurve inversion, the shape and spin of the asteroid as well as its scattering properties are solved for. Conventionally, the shape model is a convex polyhedron, where the free parameters are either the individual polyhedron facet areas or the coefficients of the spherical-harmonics series describing the Gaussian curvature of the surface [1]. Spin and shape models can be obtained using relative photometry by applying simple scattering laws, such as a combination of the Lommel-Seeliger and Lambert laws. Scattering properties can usually be assumed to be homogeneous over the surface. Extensive results of asteroid lightcurve inversion have been published by, e.g., Torppa et al. [2]. Deriving the scattering parameters of more complicated scattering laws constitutes a challenge. As an inverse problem, it is not as stable as plain spin and shape determination, and improvements in the available scattering models are called for (e.g., [3, 4]).

In Sect. 2, we describe the main features of the current simplex algorithms aimed at statistical inversion of asteroid spins, shapes, and scattering properties (see also [5]). We present some first results in Sect. 3, and Section 4 contains the conclusions and future prospects.

2 Simplex inversion

Whereas conventional lightcurve inversion consists of two parts, that is, the derivation of the normal-vector distribution and the subsequent derivation of the convex shape from the normal vectors, in simplex inversion, the convex shape solution is directly searched for. There are four parameters for the spin characteristics: the rotational period, the ecliptic longitude and latitude of the rotational pole, and the rotational phase of the object at a given time. The shape is specified using triangles with the Cartesian coordinates of the nodes as free parameters. Altogether, there are $3 + 3N$ free parameters where N is the number of nodes, the rotational phase becoming redundant because of the general shape model. The initialization of the simplex can be accomplished, e.g., by using prolate spheroids. For a detailed description of downhill simplex minimization, the reader is refered to Press et al. [6].

The simplex minimization allows for a flexible incorporation of conditions on the shape to be searched for. At each iteration step, the convexity of the shape is verified, returning a rejection for concave solutions. Solutions are constrained into a realistic regime in radial distances, that is, only radial distances within [0.3, 1.0] are presently accepted. Solutions are further constrained by the requirement that the triangle mesh be mathematically well defined during the minimization.

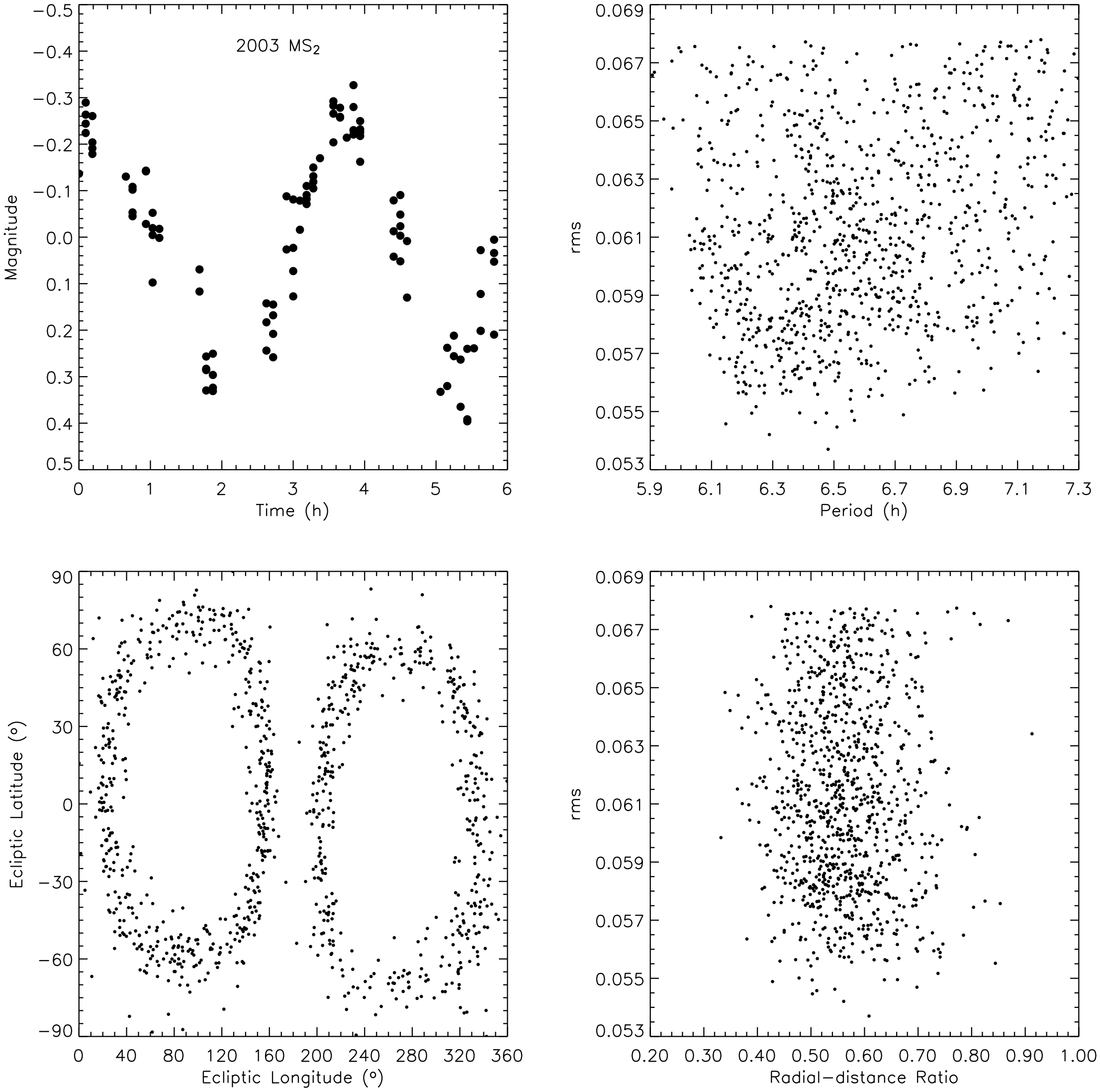

Figure 1: The original lightcurve observations (top left), rotational periods vs. rms (top right), pole longitudes vs. latitudes (bottom left), and ratios of minimum and maximum nodal radial distances vs. rms (bottom right) for the near-Earth object 2003 MS$_2$ as obtained by simplex inversion.

3 Results and discussion

We have applied the simplex inversion methods to the limited lightcurve observations of the near-Earth objects 2003 MS$_2$ and (1981) Midas, and to the extensive observations of (1580) Betulia. In all cases, we have succeeded in obtaining realistic spin and convex shape solutions within reasonable computing times.

In Fig. 1, we depict a distribution of 1000 sample solutions for 2003 MS$_2$ based on the single lightcurve observed at the Nordic Optical Telescope (NOT) [5]. The inverse problem entails the derivation of three spin parameters and 42 shape parameters, the Cartesian coordinates of the 14 nodes in the discretized shape (there are 24 triangular facets involved). The low-resolution characterization of the shape is justified by the limited data and the success of the inverse method in yielding acceptable fits to the data (at best, the rms difference is 0.054). Simplex inversion indicates forbidden regions in the pole orientation and a ratio of minimum-to-maximum nodal radial distances within 0.55 ± 0.0.15.

Figure 2 shows four sample shapes for (1981) Midas based on altogether eight lightcurves [8, 7, 5] spanning 18 years with three differing illumination and observation geometries. Using 54 shape parameters (32 triangles), the rms values of the sample solutions varied from 0.048 (top left) to 0.055 (bottom right). For (1580) Betulia, using the same number of parameters, we have obtained tentative solutions with rms values of 0.05-0.06.

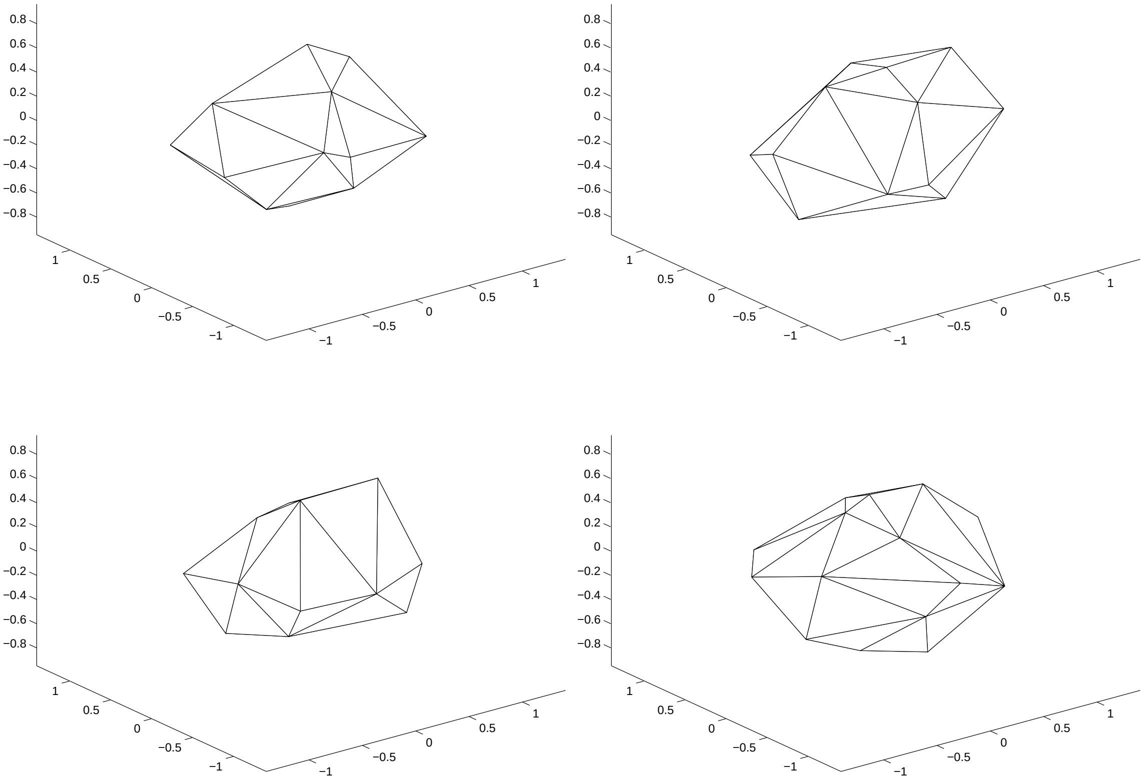

Figure 2: Four sample shapes for the near-Earth object (1981) Midas as derived by simplex inversion.

4 Conclusions

We have developed simplex inversion methods for deriving asteroid spins, shapes, and scattering properties from photometric lightcurve observations using general convex shapes. With the help of the novel methods, we have successfully assessed both limited and extensive lightcurve observations of three near-Earth objects. In the future, we will compare the simplex and conventional inversion methods, and plan to apply the methods to the forecoming sparse photometric observations by the ESA Gaia mission (launch in 2011).

Acknowledgments

We thank the Nordic Optical Telescope (NOT). Most of the data utilized have been obtained using ALFOSC, which is owned by the Instituto de Astrofisica de Andalucia (IAA) and operated at the Nordic Optical Telescope under agreement between IAA and the Astronomical Observatory of Copenhagen. Research supported, in part, by the Academy of Finland.

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