A FVM BASED ON A CELL VERTEX SCHEME FOR THE SOLUTION OF THE RTE IN 3D COMPLEX GEOMETRIES

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NUMERICAL SCHEME

FVM applied to the RTE

The angular discretization associated to the RTE is uniform and the spatial domain of interest is divided into four-node tetrahedron elements. All dependent variables are stored at the nodes of the mesh, and the equation for each variable is obtained from its discretized conservation equation written for a V_P control volume surrounding node P. The discretized RTE (for a nonscattering gray medium) over V_P and a $\Delta\Omega^k$ discrete solid angle associated to the $\mathbf{\Omega}$ direction gives [1,2]:

$$\sum_{f=1}^{N_f} A_f \; I_{i_f}^k \int_{\Delta\Omega^k} (\mathbf{\Omega}\cdot\mathbf{n_f}) \; d\Omega \;=\; \kappa_p \left\{ \, I_b(T_p) - I_p^k \, \right\} \; \Delta\Omega^k \;\; V_p \tag{1}$$

To solve the set of equations, closure relations are needed between the integration-point values $I_{i_f}^k$ and the nodal values of the radiation intensity.

Closure relations

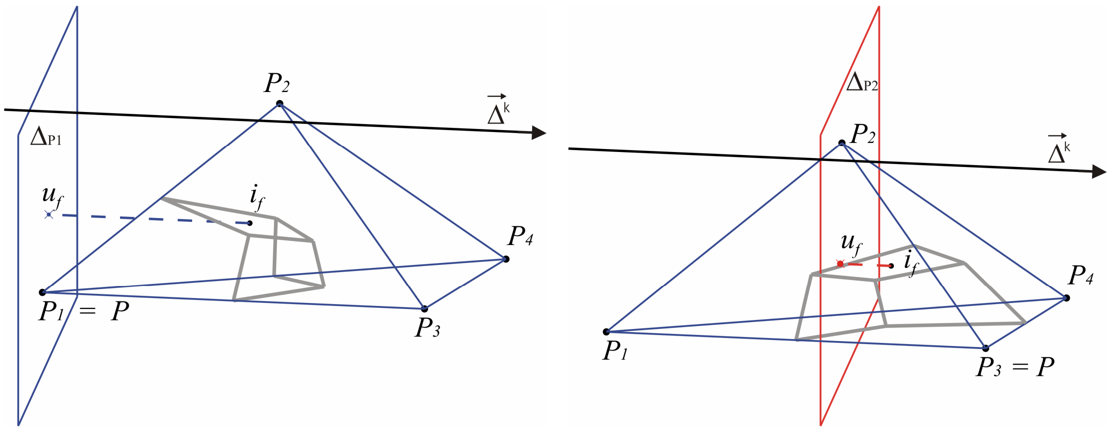
For a temperature and an absorption coefficient constant in the medium, the closure relation of exponential type [1] is given by:

$$I_{i_f}^k = I_{u_f}^k \; \exp(-\kappa \Delta_{S_f}) + I_b(T_P)\,(1-\exp(-\kappa \Delta_{S_f})) \tag{2}$$

where u_f and i_f are on the same optical way of $\mathbf{\Omega}^k$ discrete direction (u_f being located upstream from i_f and Δ_{Sf} is the distance between points i_f and u_f). For a given V_P control volume, the location of point u_f depends on the form of the (P_1, P_2, P_3, P_4) tetrahedron and the position of point i_f in relation to node P (as illustrated in figure 1). Thus, 2 cases have to be considered:

- case 1 (figure 1a): when i_f is located downstream from node P (here equal to node P_1), i_f is projected in a point u_f in the plane ΔP_1;
- case 2 (figure 1b): when i_f is located upstream from node P (here equal to node P_3), i_f is projected in a point u_f in the plane ΔP_2;

where ΔP_1 and ΔP_2 are the planes orthogonal to the $\mathbf{\Omega}^k$ discrete direction that pass respectively by the nodes P_1 and P_2 . In this work, I_{uf}^k is approximated (using only one node of interpolation) by $I_{P_1}^k$ (case 1) or $I_{P_2}^k$ (case 2). This scheme is simpler to implement. Thereafter, we plan to improve it by projecting the integration points i_f on one of the faces of the tetrahedron. In this way, I_{uf}^k will be interpolated from the three nodes that define one face of the tetrahedron.



(a) i_f located downstream from node P *(b) i_f located upstream from node P*

Figure 1 : Partial volume associated with node *P* in a tetrahedron

RESULTS AND DISCUSSION

Two test cases are presented for a nonscattering gray medium with black walls. The first test case deals with a unit cubic cavity (figure 2) taken from [3,4]. The temperature of the medium is constant and equal to 100K. The temperatures of the walls are cold (equal to 0K). Our calculations have been carried with 1,332 nodes (mesh 1) and 2,457 nodes (mesh 2). (8×4) discrete directions (24 azimuthal directions and 3 polar directions) have been used. Figure 3 shows the dimensional incoming radiative heat flux along the centerline position (x = 0.5) of the top wall for three values of the absorption coefficient. It can be seen that our results are in agreement with results reported in the literature, showing the validity of our numerical method in this first test case. The second test case (taken from [3,5]) deals with a *L*-shaped enclosure (figure 4). The temperature of the medium is constant and equal to 1,000K. The temperatures of the walls are equal to 500K. Our calculations have been carried with 848 nodes (mesh 1) and 2,292 nodes (mesh 2). The angular grid has been constructed using (6×4) discrete directions. Figure 5 shows the incoming radiative heat flux along the A-C axis, for three values of the absorption coefficient. The results obtained are compared with those reported in the literature. We observe a few discrepancies with the reference solutions, our solution being closer with the finer space grid. We think that these discrepancies are due to our numerical scheme, which currently uses only one interpolation node.

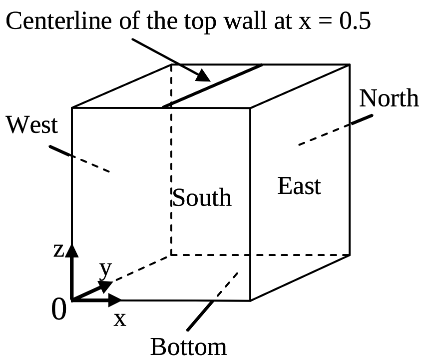


Figure 2 : Cubic cavity

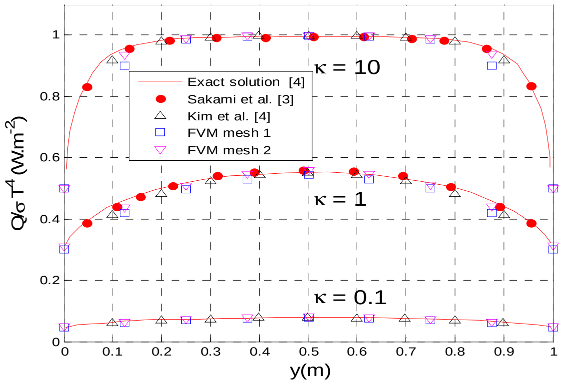


Figure 3 : Dimensional incoming radiative heat flux along the centerline of the top wall at x = 0.5

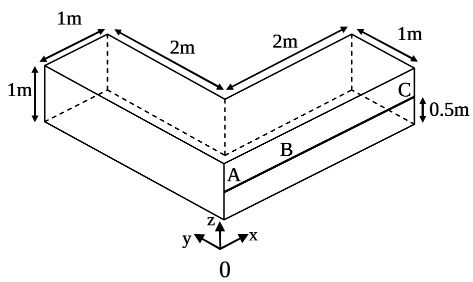


Figure 4 : L- shaped enclosure

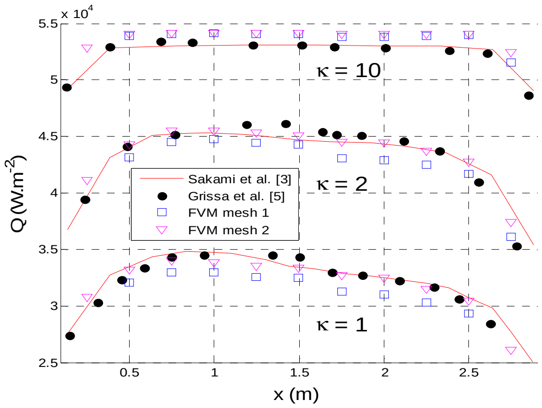


Figure 5 : Incoming radiative heat flux along A-C axis

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