Numerical Simulation of Turbulent Natural Convection and Gas Radiation in Differentially Heated Cavities Using FVM, DOM and LES

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INTRODUCTION

Natural convection in differential heated cavities has been widely studied experimentally and numerically. However, experimental results usually differ from numerical solutions. Since in most of those experimental enclosures absolute temperatures of the walls are relatively high, it has been traced that these discrepancies could be due to surface or surface and gas radiation. Nevertheless, few studies have been done taking into account only surface radiation (non-participating medium) and much less taking into account surface and gas radiation (participating medium). When gases contained inside the cavity are participating in radiation the governing equations of motion of the fluid are coupled to the integro-differential equation of radiative transport (RTE) through a new term, the divergence of radiative heat transfer, affecting the fluid flow.

The aim of the present work is to analyse the effect of surface radiation as well as both surface and gas radiation on the turbulent natural convection in a tall differentially heated cavity. Standard RANS methods and advanced LES formulations are used.

MATHEMATICAL FORMULATION AND NUMERICAL METHOD

The turbulent flow is described by means of Large Eddy Simulation (LES) and Regularization Modelling (RGM) using symmetry-preserving discretizations[1].

The spatial filtered Navier-Stokes equations,

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \left(\overline{u}_i \overline{u}_j\right)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)\right] - \frac{\partial \tau_{ij}}{\partial x_j} - \beta\left(\overline{T} - T_o\right) g_i \tag{1}$$

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \tag{2}$$

where

$$\tau_{ij} = -2\nu_{sgs}\overline{S}_{ij} + \frac{1}{3}\delta_{ij}\tau_{kk} \tag{3}$$

and

$$\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) \tag{4}$$

the subgrid-scale viscosity, ν_{sgs}, is modelled[2] in order to stop the production (by means of vortex stretching) of scales $< \Delta$: $\nu_{SGS} = c^2\Delta^2 r^+/q = C_S^2\Delta^2|\overline{S}|$ where $C_S = \frac{c^2 r}{2\sqrt{q^3}}$, $|\overline{S}| = \left(2\overline{S}_{ij}\overline{S}_{ij}\right)^{1/2}$, $r = -\det(S_{ij})$ and $q = \frac{1}{4}|\overline{S}|^2$.

The filtered temperature transport equation is

$$\frac{\partial \overline{T}}{\partial t} + \frac{\partial\left(\overline{u}_j\overline{T}\right)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\lambda_{\text{eff}}\frac{\partial \overline{T}}{\partial x_j}\right] - \frac{1}{\rho c_p}\overline{\nabla \cdot q_{rad}} \tag{5}$$

where $\lambda_{\text{eff}} = \nu/Pr + \nu_{sgs}/Pr_t$. The $\nabla \cdot q_{rad}$ is obtained from the RTE. In the present study RTE is solved for an emitting-absorbing nonscattering grey medium:

$$\frac{dI}{ds} = -\kappa I + \kappa I_b \tag{6}$$

The filtered RTE is

$$\frac{d\overline{I}}{ds} = -\overline{\kappa I} + \overline{\kappa I_b} = -\overline{\kappa}\overline{I} - \left(\overline{\kappa I} - \overline{\kappa}\overline{I}\right) + \overline{\kappa}\overline{I_b} + \left(\overline{\kappa I_b} - \overline{\kappa}\overline{I_b}\right) \approx -\overline{\kappa}\overline{I} + \overline{\kappa}\overline{I_b} \tag{7}$$

The simplest way to close the filtered RTE is used, i.e. the terms in parenthesis are ignored and $\overline{I_b} \approx I_b(\overline{T})$. In a similar way it was done in the previous work[3]. The filtered radiative source term is approximated using the same criteria as in the filtered RTE,

$$\overline{\nabla \cdot q_{rad}} = 4\pi\,\overline{\kappa I_b} - \int_{4\pi}\overline{\kappa I}d\omega \approx 4\pi\,\overline{\kappa}\overline{I_b} - \int_{4\pi}\overline{\kappa}\overline{I}d\omega \tag{8}$$

Radiation is simulated using the Finite Volume Method (FVM) or the Discrete Ordinates Method (DOM)[4].

Numerical results are carried out by using the CFD code TERMOFLUIDS which is an intrinsic 3D parallel CFD code applied to unstructured meshes[5]. In all cases, a explicit finite volume fractional-step based algorithm is used. The pressure equation is solved by means of a parallel FFT Schur decomposition solver. Discretized algebraic radiative transfer equations are solved using the parallel iterative solver GMRES or a parallel sweep solver.

STUDIED CASES

The studied case corresponds to a cavity of aspect ratio 5:1 filled with air and a Rayleigh number based on the height of $4.48 \cdot 10^{10}$, a Planck number of 1253.8 and a ratio of temperature of 0.1399. The four walls are considered black. The analysis of participating medium has been performed for an optical thickness ($\tau = \overline{\kappa}H$) of 1.0.

Simulations are compared with experimental data available in the literature[6] and the previous numerical results obtained by the authors using $k-\epsilon$ and $k-\omega$ low-Reynolds number two-equation eddy-viscosity turbulent models[3]. Figure 1 shows dimensionless turbulent viscosity field and iso-temperature lines using RANS method. In the congress detailed results will be presented.

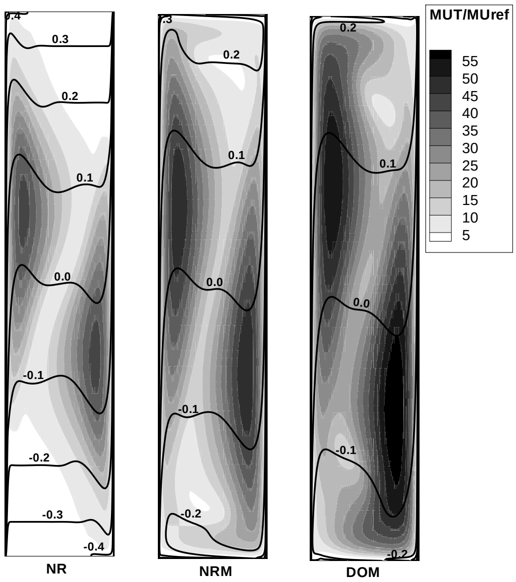

Figure 1: Dimensionless turbulent viscosity field and iso-temperature lines using PDH model[7] at three radiative situations (NR: No Radiation; NRM: Net Radiation Method; DOM: participating medium) .

REFERENCES

[1] C. D. Pérez-Segarra, O. Lehmkuhl, J. Jaramillo, G. Colomer, and A. Oliva. Turbulence modelling and numerical issues: from rans to dns and les. In *Actas da III Conferência Nacional em Mecânica de Fluidos, Termodinâmica e Energia (MEFTE - BRAGANÇA 09) p3 paper (in CD)*, pages 1–31, 2009.

[2] R. Verstappen. When does eddy viscosity damp subfilter scales sufficiently? In *Quality and Reliability of Large-Eddy Simulations II*, Pisa, Italy, September 2009.

[3] R. Capdevila, CD. Pérez Segarra, G. Colomer, and A. Oliva. Numerical Simulation of Turbulent Natural Convection and Gas Radiation in Differentially Heated Tall Cavities Using Discrete Ordinates Method. In *Proceedings of the V International Symposium on Turbulence, Heat and Mass Transfer*, pages 535–538, 2006.

[4] R. Capdevila, C. D. Pérez-Segarra, and A. Oliva. Development and comparison of different spatial numerical schemes for the radiative transfer equation resolution using three-dimensional unstructured meshes. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 111(2):264–273, 2010.

[5] O. Lehmkuhl, C. D. Perez-Segarra, R. Borrell, M. Soria, and A. Oliva. TERMOFLUIDS: A new Parallel unstructured CFD code for the simulation of turbulent industrial problems on low cost PC Cluster. In *Proceedings of the Parallel CFD 2007 Conference*, pages 1–8, 2007.

[6] R. Cheeswright, K.J. King, and S. Ziai. Experimental Data for the Validation of Computer Codes for the Prediction of Two-dimensional Buoyant Cavity Flows. In *Proceedings of Significant Questions in Buoyancy Affected Enclosure or Cavity Flows*, pages 75–81, 1986.

[7] S. Peng and L. Davidson. Computation of Turbulent Buoyant Flows in Enclosures with Low-Reynolds Number $k-\omega$ Models. *International Journal of Heat and Fluid Flow*, 20:172–184, 1999.