ANALYTICAL SOLUTION OF P1 APPROXIMATION IN RECTANGULAR ENCLOSURE WITH GRAY MEDIUM

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ABSTRACT

Radiative heat transfer in participating medium has considerable importance in manufacturing, combustion and energy systems. The radiative transfer equation is an integro-differential equation with direction, wavelength, position and time dependent properties and variables. The solution of the radiative transfer equation often necessitates utilizing specialized methods that rely on representing the transfer equation in integral or differential form. A detailed list and classification of these methods are presented in [1].

Representation of radiative transfer in differential form is often preferable when its solution is sought in conjunction with continuity, momentum and energy equations that are all in differential form. Moreover, as such a solution will necessitate iterations, the solution efficiency is also important. One method of representing radiative transfer equation in differential form is spherical harmonics or PN approximation where intensity field is expressed as a series. The single term approximation is referred as P1 method, whereas 3 term approximation is referred as P3 method.

PN approximation is widely investigated in the literature. Davison [2] used PN approximation to model neutron transport. For thermal radiation problems, Modest [3] presented analytical solutions for absorbing-emitting medium in two dimensional rectangular enclosure with gray walls by using P1 method. The application of this study was extended for absorbing-emitting and anisotropic scattering medium between plane parallel medium in [4]. A two dimensional enclosure with absorbing-emitting and isotropic scattering medium was studied by Ratzel [5]. Similarly, Ratzel and Howell [6] applied P1 and P3 methods for two dimensional absorbing-emitting medium with heat generation. Mengüç [7] extended two dimensional analysis to three dimensional, considering absorbing, emitting and anisotropically scattering medium. In the studies of Mengüç [7] and Ratzel [5], numerical calculations are used to verify presented analytical solutions. Wu [8] compared standard differential approximation, Olfe's modified differential approximation and exact integral formulation for a rectangular enclosure with absorbing, emitting and scattering medium with linear anisotropy. Furthermore, Szu-Cheng and Liou [9] generalized the spherical harmonics method for different coordinate systems. The common conclusion of these studies is that P1 approximation presents acceptable accuracy in optically thick medium and its computational efficiency is high.

The goal of this study is to present a generalized analytic expression for that can be implemented easily when solution of radiative heat transfer in two dimensional rectangular enclosure problems are sought. In present study, general analytical expressions for P1 approximation are derived for emitting-absorbing and linearly anisotropic scattering gray medium in radiative equilibrium, in a rectangular

enclosure as shown in Figure 1. Medium is considered as gray and homogeneous and the surfaces are diffuse, gray with all properties are considered independent of temperature. The earlier studies such as [3-7] have presented similar solutions for specific boundary conditions. The solutions in this study are derived for generic boundary conditions that can be considered for constant surface temperature or surface heat flux.

The phase function for linearly anisotropic scattering can be presented as

$$\Phi(\hat{\mathbf{s}},\hat{\mathbf{s}}')=1+M\,\hat{\mathbf{s}}\cdot\hat{\mathbf{s}}' \tag{1}$$

The radiative transfer equation can be simplified with P1 approximation as

$$\nabla_\tau G=\nabla_\tau I^{(0)}=-(3-M\,\omega)\boldsymbol{q} \tag{2}$$

For a system at steady state, with medium at radiative equilibrium the equation required to be solved for intensity distribution in the medium is

$$\frac{\partial^2 I^{(0)}}{\partial\tau_x^2}+\frac{\partial^2 I^{(0)}}{\partial\tau_y^2}=0 \tag{3}$$

The resulting differential equation is Laplace's equation and its solution is well established. Marshak's boundary condition is used to solve the problem. The generic boundary conditions for this type of problem can be presented as

$$\pm A_i\frac{\partial I^{(0)}}{\partial\tau_j}+I_i^{(0)}=C_i \tag{4}$$

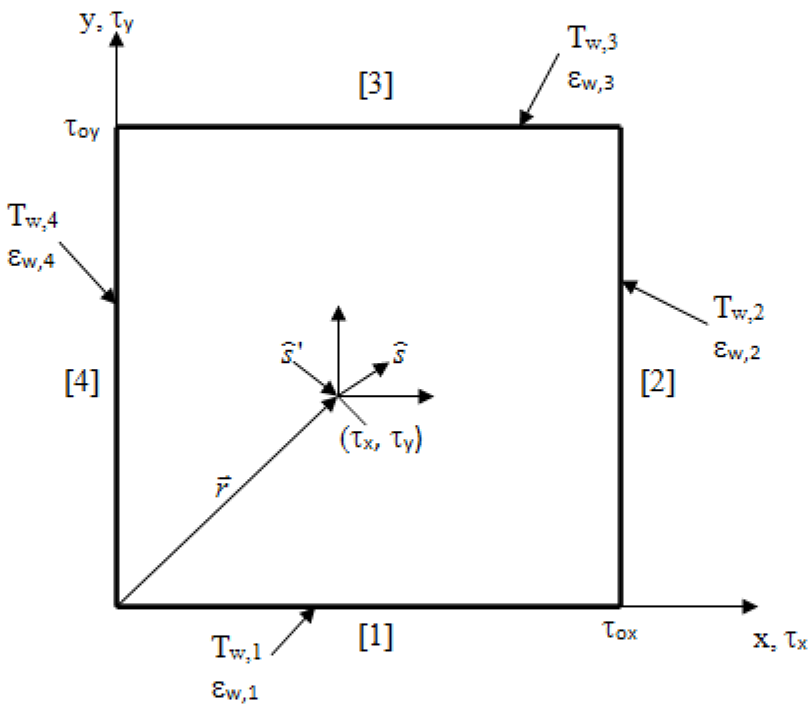

Figure 1. Schematic of the rectangular enclosure

Here,

$$A_i = \frac{4 - 2\varepsilon_{w,i}}{(3 - M\omega)\varepsilon_{w,i}} \tag{5}$$

$$C_i = 4\sigma T_{w,i}^4 \tag{6}$$

$$i = 1, 2, 3, 4 \tag{7}$$

The positive sign for the first term at right hand side of equation (5) is used for surfaces 2 and 3, while the negative sign is used for surfaces 1 and 4. Besides, j represents x for surfaces 2 and 4 and y for surfaces 1 and 3.

The resulting equations are compared with solutions presented in literature for non-scattering, isotropic scattering and linearly anisotropic scattering medium with various boundary conditions. The results show consistency with precedent studies.

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