RAD-13-P14

THE GENRALIZED SLW METHOD

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The Spectral Line Weighted-sum-of-gray-gases (SLW) method is the first approach for spectral modeling of radiative transfer in high temperature gases developed on the basis of the high resolution gas absorption spectrum. Since that time the method has undertaken many extensions, interpretations and variations in the form of the ADF, FSK, CW and MBWSGG methods, and also in the form of hybrid of the SLW method with other spectral models such as EWBM, reordered wavenumber, correlated k-distributions, etc. With increase of the number of gray gases, the SLW model approaches the exact model.

The main tool of the SLW method is the novel distribution function $F\left(C,T_g,T_b\right)$ termed the Absorption Line Blackbody Distribution Function (ALBDF) which is the characterization of the high resolution gas absorption cross-section $C_\eta\left(T_g\right)$ at gas temperature T_g regarding the Planck spectral distribution of the blackbody emissive power of radiation at the source temperature T_b.

The present work is the unified formulation of the weighted sum of gray gases approach to spectral modeling of gas radiation based on alternative application of both the direct ALBDF and of the inverse ALBDF. This yields two independent but completely equivalent symmetric gray gas spectral models which make the SLW method universal, efficient, and flexible. It allows the resolution of some difficulties which appear when the methods are applied individually, such as inversion of the distribution function or solution of the implicit equations in applications to non-isothermal gas medium.

The ALBDF $F\left(C,T_g,T_b\right)$ is defined as a function of the variable C, which defines the fraction of the total blackbody emissive power $E_b\left(T_b\right)=\sigma T_b^4$ emitted at the source temperature T_b is in the part of the spectrum where the gas absorption cross-section $C_\eta\left(T_g\right)$ is below the prescribed value C (see Fig. 1).

The Inverse ALBDF $C\left(F,T_g,T_b\right)$ is defined as a function of the variable F, which reveals the value of the absorption cross-section C, for which the fraction of the blackbody emissive power $E_b\left(T_b\right)=\sigma T_b^4$ emitted at temperature T_b that is in the part of the spectrum where the gas absorption cross-section $C_\eta\left(T_g\right)$ is below C, is equal to prescribed value F. Existing correlations or a look-up table of the ALBDF can be used for inversion of the ALBDF in analytical form.

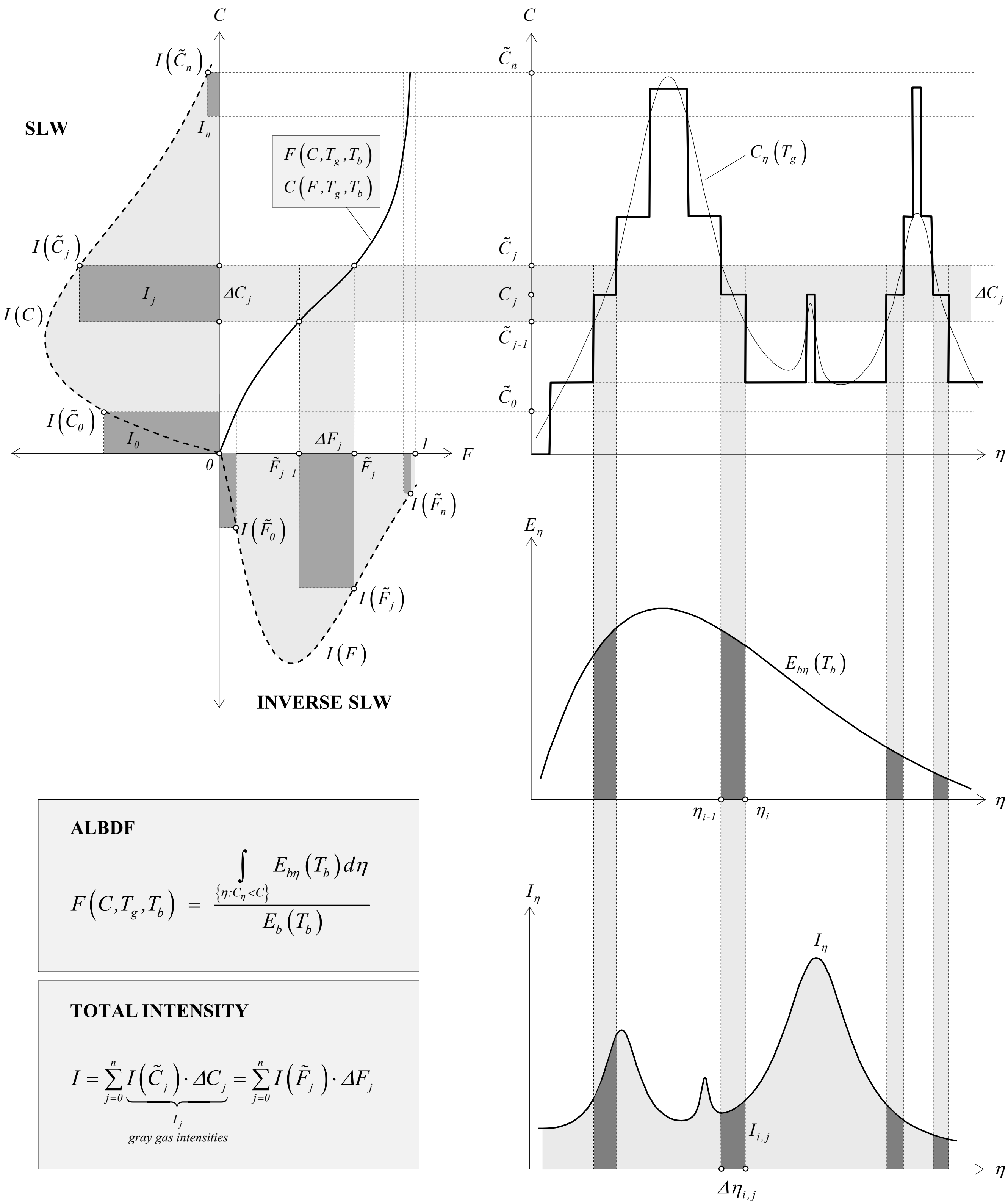

Figure 1. Geometrical Interpretation of the SLW Method and the Inverse SLW Method

The SLW spectral model with spectral "windows" is based on subdivision of the C variable with the help of supplemental cross-sections $\tilde{C}_0, \tilde{C}_1, ..., \tilde{C}_n$ chosen between $\tilde{C}_0 = C_{min}$ and $\tilde{C}_n = C_{max}$ in the absorption cross-section: $\tilde{C}_j = \tilde{C}_{j-1} + \Delta C_j$, j =1,2,…,n. The gray gas absorption cross sections are defined with the discrete cross-sections $\tilde{C}_{j-1} < C_j < \tilde{C}_j$, and the value $C_0 = 0$ – corresponding to a clear gas ("windows"). This construction yields a piece-wise histogram spectrum: $\kappa_\eta = \kappa_j = NYC_j$ for η such that $\tilde{C}_{j-1} < C_\eta(T_g) < \tilde{C}_j$, and $\kappa_\eta = 0$ for η such that $C_\eta(T_g) < \tilde{C}_0$.

Subdivision of the cross-section interval is usually performed with evenly logarithmically spaced subintervals $\tilde{C}_j = \tilde{C}_0 \left(\tilde{C}_n / \tilde{C}_0 \right)^{j/n}$, $j = 0, 1, ..., n$, accompanied by computation of the gray gas absorption cross-section as a geometric mean $C_j = \sqrt{\tilde{C}_{j-1} \tilde{C}_j}$. The optimized subdivision into supplemental cross-sections is also sometimes used. The minimal SLW spectral model with "windows" consists of a single gray gas and of a single clear gas (the optimized SLW-1 Method).

The SLW spectral model without "windows" is based on subdivision of F into subintervals $\tilde{F}_0 = 0, \tilde{F}_1, ..., \tilde{F}_{n-1}, \tilde{F}_n = 1$ with the discrete values $\tilde{F}_{j-1} < F_j < \tilde{F}_j$. Then the gray gas absorption coefficients can be defined as $\kappa_j = NYC(F_j, T_g, T_b)$.

Construction of the model without "windows" is usually based on integral quadratures. The method of moments can be used for partitioning the F variable for construction of a model with "windows."

The minimal spectrum model without "windows" is a gray gas model.

Exact Limit. With an increase in the number of gray gases n, the histogram spectrum of both models approaches the continuous "exact" high resolution absorption spectrum $C_\eta(T_g)$.

The SLW Method – calculation of the total intensity: $n \to \infty$ **Exact Limit**

$$I = \sum_{j=0}^{n} \sum_{i} \int_{\Delta\eta_{i,j}} I_\eta d\eta = \sum_{j=0}^{n} \sum_{i} I_{j,i} = \sum_{j=0}^{n} \left(\frac{\sum_i I_{i,j}}{\Delta C_j} \right) \Delta C_j = \underbrace{\sum_{j=0}^{n} I(\tilde{C}_j) \cdot \Delta C_j}_{I_j} \quad \to \quad I = \int_0^\infty I(C) dC$$

The Inverse SLW Method – calculation of the total intensity:

$$I = \sum_{j=0}^{n} \sum_{i} \int_{\Delta\eta_{i,j}} I_\eta d\eta = \sum_{j=0}^{n} \sum_{i} I_{j,i} = \sum_{j=0}^{n} \left(\frac{\sum_i I_{i,j}}{\Delta F_j} \right) \Delta F_j = \underbrace{\sum_{j=0}^{n} I(\tilde{F}_j) \cdot \Delta F_j}_{I_j} \quad \to \quad I = \int_0^1 I(F) dF$$

In the SLW method, the Radiative Transfer Equations for the gray gas intensities are

$$\frac{\partial I_j}{\partial s} = -\kappa_j I_j + a_j \kappa_j I_b, \qquad \text{where} \quad a_j = \Delta F_j .$$

In the Inverse SLW Method, the RTEs for the gray gas intensities are

$$\frac{\partial I(\tilde{F}_j)}{\partial s} = -\kappa_j I(\tilde{F}_j) + \kappa_j I_b$$

With increase of the number of gray gases, both methods approach the Exact Limit.