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SUBGRID SCALE MODEL FOR RADIATIVE TRANSFER IN OPTICALLY THICK PARTICIPATING MEDIA

Laurent Soucasse, Philippe Rivière and Anouar Soufiani
CNRS, UPR 288, Laboratoire EM2C, F-92290 Châtenay Malabry, France
École Centrale Paris, F-92290 Châtenay Malabry, France

EXTENDED ABSTRACT

Introduction Numerical simulation of coupled convection and radiation heat transfer in absorbing fluids is particularly difficult when the involved flow is turbulent and when all turbulence length scales are not optically thin. Earlier studies dedicated to radiation in turbulent flows mainly focus on the modeling of radiation non-linearities and their effects on Reynolds Averaged modeling of velocity, concentration of absorbing species and temperature fields [1, 2]. Although 3D Direct Numercial Simulations (DNS) of turbulent flows become more and more practicable, 3D reference radiative calculations, able to capture all turbulence length scales, remain computationally too expensive. To overcome this difficulty, we propose in this work a model based on the filtering of the radiative source term. The filtered part is treated by the reference ray tracing method and the remaining scales are treated by the approximate SP_3 method. This approach can be coupled with a DNS or a Large Eddy Simulation of a turbulent flow.

Spatial Filtering We consider a gray non scattering semi-transparent medium of constant absorption coefficient κ. First, the emission source term $S(\mathbf{r}) = \kappa\, I_b(\mathbf{r})$ is filtered by truncating the 3D Fourier transform $\hat{S}(\mathbf{k})$ of $S(\mathbf{r})$ at a cutoff wavenumber k_c. This filtering leads to the decomposition $S(\mathbf{r}) = \overline{S}(\mathbf{r}) + S'(\mathbf{r})$. Due to the linearity of the Radiative Transfer Equation (RTE), the intensity field is then splitted into a response $\widetilde{I}(\mathbf{r},\mathbf{u})$ to the filtered part $\overline{S}(\mathbf{r})$ and a response $I''(\mathbf{r},\mathbf{u})$ to the subgrid part $S'(\mathbf{r})$ according to

$$I(\mathbf{r},\mathbf{u}) = \widetilde{I}(\mathbf{r},\mathbf{u}) + I''(\mathbf{r},\mathbf{u}). \tag{1}$$

One can then calculate separately the filtered and subgrid radiative powers following

$$P(\mathbf{r}) = \widetilde{P}(\mathbf{r}) + P''(\mathbf{r}) = \left(4\pi\,\overline{S}(\mathbf{r}) - \kappa \int_{4\pi} \widetilde{I}(\mathbf{r},\mathbf{u})\, d\Omega\right) + \left(4\pi\, S'(\mathbf{r}) - \kappa \int_{4\pi} I''(\mathbf{r},\mathbf{u})\, d\Omega\right). \tag{2}$$

The filtered radiative power is calculated, using the accurate ray tracing method. A coarser mesh than that required to obtain the complete field $P(\mathbf{r})$ can be used to discretize the filtered RTE. The subgrid radiative power is calculated using the following approximate subgrid model.

Subgrid model Applying the SP_3 approximation [3] to the subgrid RTE leads to:

$$-\nabla \cdot \left(\frac{\mu_i}{\kappa} \nabla \psi_i(\mathbf{r}) \right) + \kappa\, \psi_i(\mathbf{r}) = 4\pi\, S'(\mathbf{r}), \qquad i = 1, 2 \tag{3}$$

$$P''(\mathbf{r}) = -\nabla \cdot \frac{1}{\kappa} \nabla \left(a_1\, \psi_1(\mathbf{r}) + a_2\, \psi_2(\mathbf{r}) \right), \tag{4}$$

where a_i and μ_i are constant coefficients. In Fourier space, the solution of these equations is straightforward and we obtain for $k > k_c$:

$$\hat{\psi}_i(\mathbf{k}) = \frac{\kappa}{\mu_i\, k^2 + \kappa^2} 4\pi\, \hat{S}(\mathbf{k}), \qquad i = 1, 2 \tag{5}$$

$$\hat{P}''(\mathbf{k}) = \left(\frac{a_1}{\mu_1 + (\kappa/k)^2} + \frac{a_2}{\mu_2 + (\kappa/k)^2} \right) 4\pi\, \hat{S}(\mathbf{k}). \tag{6}$$

As the Fourier transform $\hat{S}(\mathbf{k})$ is required to calculate the filtered source term $\overline{S}(\mathbf{r})$ in the ray tracing method, the CPU cost to compute $\hat{P}''(\mathbf{k})$ is negligible and an inverse Fourier transform enables us to obtain the subgrid radiative power $P''(\mathbf{r})$. Note that the boundary conditions disappear in Fourier space which means that all the subgrid quantities are assumed to be periodic. This assumption will have small effects on the results for optically thick media, for which the SP_3 approximation is accurate, and relatively far from the walls. Anyway, the radiative transfer between the semi-transparent medium and the possible opaque walls is assumed to be well captured by the ray tracing method.

Results In order to assess the accuracy of this model, we perform radiative transfer calculations in a cubic box, of side length L, containing an absorbing medium of average temperature $T = 500\,K$ and fluctuation level $T' = 50\,K$. The 3D temperature field is stochastically generated in order to statistically satisfy an exponential spatial autocorrelation function $C(r) = \exp(-r/\Lambda)$, following the method described in [4]. The walls of the box are perfectly diffuse reflecting, the optical thickness of the medium is $\kappa\, L = 25$ and the integral length scale is such that $\Lambda/L = 0.05$. Each non-filtered quantity is descritized using 320^3 spatial points and can be represented with 160^3 Fourier modes ($kL/2\pi = -80, \cdots, 79$ in each x, y or z direction). The source term is rigorously calculated as $S(\mathbf{r}) = \kappa\sigma T^4(\mathbf{r})/\pi$ and filtered at the cutoff wavenumber $k_c L/2\pi = 20$. Filtered radiative power $\widetilde{P}(\mathbf{r})$ is calculated from the filtered source term $\overline{S}(\mathbf{r})$ using 1600 directions and 80^3 spatial points and then interpolated on the finer mesh with a cubic 3D interpolation. Subgrid radiative power $P''(\mathbf{r})$ is calculated in Fourier space from the highest modes ($k > k_c$) of $\hat{S}(\mathbf{k})$ and then evaluated in physical space using inverse Fourier transform.

Figure 1 shows the total, filtered and subgrid radiative power on a partial slice in the domain. Each result is compared with the corresponding reference calculation carried out with the ray tracing method using the finest mesh. First, we can note that the filter efficiently smoothes the spatial fluctuations: the fluctuations of the filtered power are more extended than those of the total power and the extremal values are smaller. Likewise, the subgrid power is significant and its variance represents 43% of the variance of the total power. The agreement between the model and the reference calculation is good for the total power and the differences are limited to about 5%. More precisely, the normalized error on the total radiative power defined by $\sigma = \left(\sum_i (P(\mathbf{r}_i) - P^{REF}(\mathbf{r}_i))^2 / \sum_i (P^{REF}(\mathbf{r}_i))^2 \right)^{1/2}$ is equal to 0.067. As it can be seen in figure 1, the differences are mainly due to the computation of the subgrid power with the subgrid model. The model enables a huge saving of computational time by a factor roughly equal to the mesh ratio (64 in this case) because the computational cost of the 3D Fourier transform is

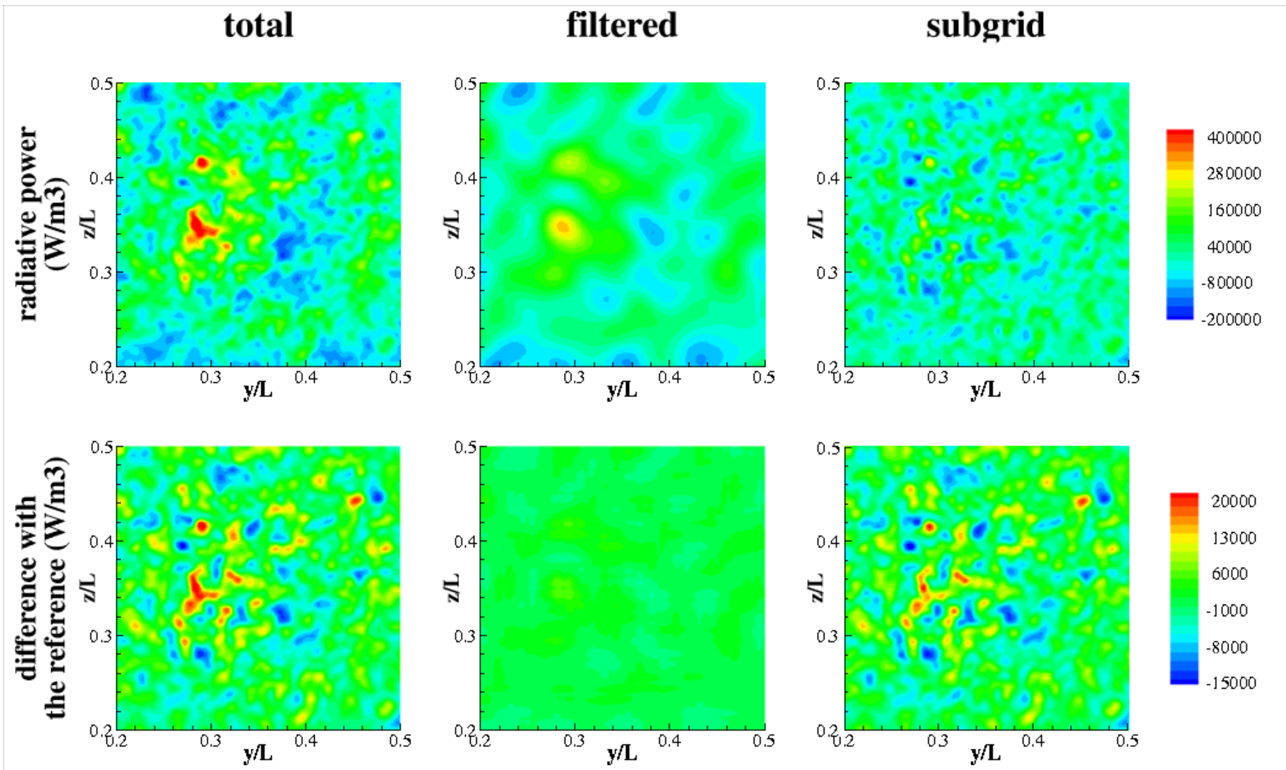

Figure 1: From left to right: total, filtered and subgrid radiative power (top) and their corresponding absolute difference compared to the reference radiative power (bottom). Slices at $x/L = 0.25$, $0.2 < y/L < 0.5$ and $0.2 < z/L < 0.5$.

very small compared to that of the ray tracing method.

Conclusion Altough the method has been presented for a gray medium, the extension to the non-gray case is straightforward if the absorption coefficient remains space independent. For space dependent radiative properties, we have tested a local correction which provides satisfactory results. Finally, this approach has been also successfully extended to non uniform grids for coupling gas radiation and turbulent boundary layer flows. More complete results will be presented during the poster session of the symposium.

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