

## FIRST MOMENT CLOSURE MODELING OF FILM COOLING EFFECTIVENESS IN SINGLE ROW OF CYLINDRICAL HOLES

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### INTRODUCTION

Making significant advances in the cooling technology requires a fundamental understanding of the physical mechanisms involved in film cooling flow fields. Most of the publications concerning film cooling of gas turbines have employed two-equation turbulent momentum flux closures combined with simple eddy diffusivity (SED) approach for turbulent heat flux modeling [e.g., Lakehal et al. 2001]. However, this approach is not always effective since there is no universal value for the turbulent Prandtl number. A higher level of turbulent heat flux modeling is developed as a remedy for some of the existing models deficiencies, in which two additional transport equations for temperature variance and its dissipation rate are considered. The combination of a two-equation momentum and a two-equation heat flux closure known as fully first-moment closure is widely used in basic flow geometries such as turbulent pipe and channel flow. Nevertheless, its application to complex flow and heat transfer processes such as film cooling is seldom addressed.

### SCOPE OF THE PAPER

The objective of the present paper is to evaluate the first-moment closure model applied to film cooling flow and heat transfer computations. The low Reynolds number  $k - \varepsilon$  turbulence model for flow field is combined with a two-equation  $k_\theta - \varepsilon_\theta$  model for thermal field to estimate the flow and heat transfer in a three-dimensional single row film cooling application. The governing equations together with the boundary conditions are solved by means of the finite volume numerical method. The test case is based on the experimental work of Sinha et al. [1991].

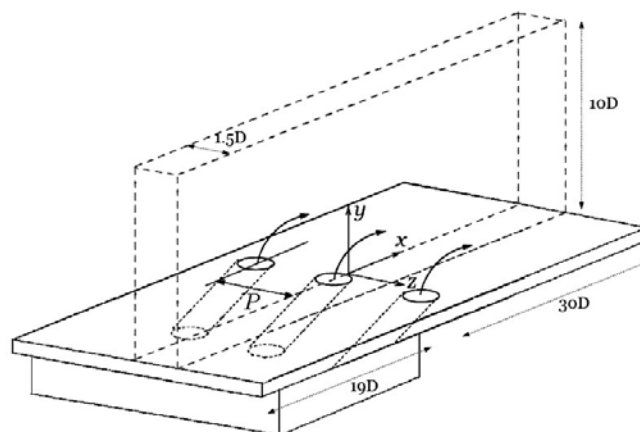


Figure 1. Schematic of the experimental setup [Sinha et al., 1991] and computational domain

It consists of a single row of jet holes on a flat surface, with a 35° streamwise injection angle and a film hole pitch-to-diameter ratio of  $P/D=3$ . The coolant fluid is injected from a supply plenum located beneath the flat surface, as shown in Fig. 1. The computational domain is extended as  $50D \times 10D \times 1.5D$  in the  $x$ ,  $y$  and  $z$  directions, respectively. In the streamwise direction ( $x$ ) the domain extends from the inflow plane located at  $19D$  upstream to outflow plane located at  $30D$  downstream of the injection hole. In the spanwise direction the domain extends from a plane through the middle of the holes ( $z/D=0$ ) to a plane at  $z/D=1.5$  in the middle between two injection holes, and symmetry conditions are imposed on these planes.

### FIRST-MOMENT CLOSURE

In the present work, it is assumed that the working fluid (air) is incompressible and Newtonian with temperature-dependent fluid properties. Turbulence effects are taken into account using the eddy viscosity/diffusivity concept. Then the constitutive closures of turbulent momentum and heat fluxes are obtained using the Boussinesq approximation. The low Reynolds number  $k - \varepsilon$  model of Chang et al. [1995] is adopted for turbulent velocity field. The set of governing equations for temperature variance and its destruction rate, based on the Deng et al.'s proposition [2001], is as follows:

$$\frac{Dk_{\theta}}{Dt} = \frac{\partial}{\partial x_j} \left[ \left( \alpha + \frac{\alpha_t}{\sigma_h} \right) \frac{\partial k_{\theta}}{\partial x_j} \right] - \alpha_t \left( \frac{\partial T}{\partial x_j} \right)^2 - \varepsilon_{\theta} \quad (1)$$

$$\frac{D\varepsilon_{\theta}}{Dt} = \frac{\partial}{\partial x_j} \left[ \left( \alpha + \frac{\alpha_t}{\sigma_{\phi}} \right) \frac{\partial \varepsilon_{\theta}}{\partial x_j} \right] - C_{p1} \sqrt{\frac{\varepsilon \varepsilon_{\theta}}{2kk_{\theta}}} \alpha_t \left( \frac{\partial T}{\partial x_j} \right)^2 - C_{d1} f_{d1} \frac{\varepsilon_{\theta}^2}{2k_{\theta}} - C_{d2} f_{d2} \frac{\varepsilon \varepsilon_{\theta}}{k} \quad (2)$$

The model functions and constants are set according to Deng et al. [2001].

### DISCUSSION OF SAMPLE RESULTS

For validation of the present proposed model, the first-moment closure model is applied in an isothermal turbulent pipe flow and the characteristics of turbulent heat transfer are considered. The temperature distribution, temperature variance, and its destruction rate are compared with the available experimental data of Hishida and Nagano [1979]. Fig. 2 shows the profiles of measured and computed mean temperature,  $T^+$ , normalized by the friction temperature in the wall coordinates.

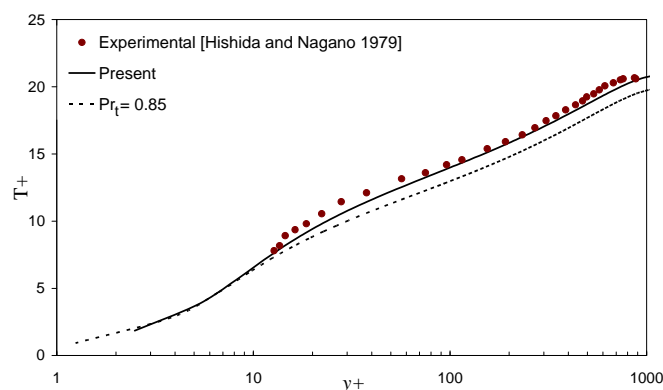


Figure 2. Normalized mean temperature profiles in wall coordinates

The present computational results are obtained using first-moment closure models for both flow and thermal fields. The present profile is in good agreement with the available experimental data of Hishida and Nagano [1979]. Fig. 3 represents the variations of center-line film cooling effectiveness,  $\eta$ , applying SED (i.e., fixed  $Pr_t=0.85$ ) and a two-equation turbulent heat flux model. Present results are compared with the existing experimental data of Sinha et al. [1991]. The SED model obviously under-predicts  $\eta$  near the injection hole ( $x/D < 7$ ) and over-predicts in the downstream region ( $x/D > 12$ ). Applying a two-equation turbulent heat flux model fairly improves the prediction of  $\eta$  in almost all  $x/D$  with a difference of less than 10% on average.

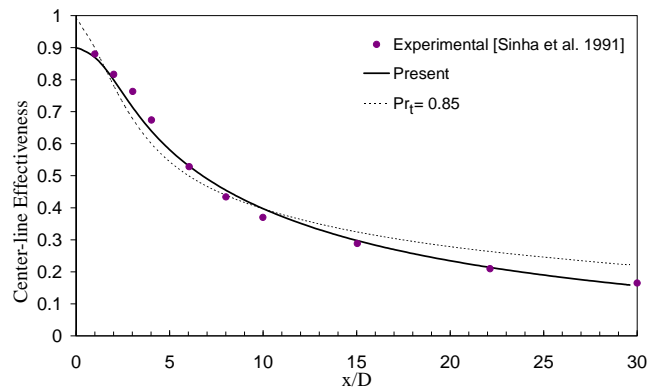


Figure 3. Variations of Center-line film cooling effectiveness

## CONCLUSION

In the present work, a first-moment closure modeling of flow and heat transfer in a single row of film cooling holes is considered. Two-equation models are applied for modeling both the Reynolds stresses tensor and turbulent heat flux vector. The turbulent heat flux model shows a significant effect on predicting the thermal field. The simple eddy diffusivity (SED) model with a constant prescribed value of  $Pr_t$  is a poor approximation for the turbulent heat flux. Applying a two-equation turbulent heat flux model fairly improves the predicted film cooling effectiveness.

## REFERENCES

- Chang, K.C., Hsieh, W.D. and Chen, C.S. [1995], A Modified Low Reynolds Number Turbulence Model Applicable to Recirculating Flow in Pipe Expansion, *Trans. ASME, J. Fluid Engr.*, Vol. 117, pp. 417-423.
- Deng, B., Wu, W. and Xi, S. [2001], A Near-wall Two-equation Heat Transfer Model for Wall Turbulent Flows, *Int. J. Heat Mass Transfer*, Vol. 44, pp. 691-698.
- Hishida, M. and Nagano, Y. [1979], Structure of Turbulent Velocity and Temperature Fluctuations in Fully Developed Pipe Flow, *ASME J. Heat Transfer*, Vol. 101, pp. 15-22.
- Lakehal, D., Theodoridis, G.S., and Rodi, W. [2001], Three-dimensional Flow and Heat Transfer Calculations of Film cooling at the Leading Edge of a Symmetrical Turbine blade model, *Int. J. Heat Fluid Flow*, Vol. 22, pp. 113-122.
- Sinha, A.K., Bogard, D.G. and Crawford, M.E. [1991], Film Cooling Effectiveness Downstream of a Single Row of Holes with Variable Density Ratio, *Trans. ASME, J. Turbomachinery*, Vol. 113, pp. 442-449.