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PASSIVE THERMAL CONTROL IN REFRIGERATION AND COOLING

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1. INTRODUCTION

The thermal control system design is configured for simple and reliable temperature control that provides the flexibility to accommodate variations in the heat load. Thermal control is achieved with minimum heat transfer among the major parts of the system by means of passive, active or hybrid methods [1].

In a physical sense, a passive thermal control (PTC) is one that uses only locally available energy sources and utilizes the natural flow paths of that energy. In other words, no auxiliary equipment (such as fans or pumps) are required to make the passive system to function - the transmission media is directed by induced convection currents or reflected/refracted to where it is needed to do the work.

Before refrigeration technology first appeared, people kept cool using natural methods: breezes flowing through windows, water evaporating from trees and fountains as well as large amounts of stone and earth absorbing daytime heat. In ancient times, kings sent servants and camels to mountain tops to gather ice. The ice was insulated with dried grasses and hauled back down to a special place directly above the king's throne where air, cooled by the ice, would descend to provide comfort for the king. The practice of cooling with nature's ice persisted even after the first thermodynamic refrigeration cycles made their debut in the mid to late 1800's. Through centuries man has always made use of a great deal of cleverness to keep cool in hot climates and seasons. He has developed a broad range of passive cooling techniques in various parts of the world up to a very impressive level of maturity: cliff dwellings through the world (ground cooling), wind towers in Iran (convective and mass cooling), sprinkling water with fountains (evaporative cooling), and white-wash (sun protection) in Southern Europe and North Africa [3]. All these cooling techniques were based on careful design in which heat and mass transfer principles did not make use of any mechanical energy that requires complicated refrigeration systems [4-7]. Hence, by employing passive cooling techniques you can often eliminate the need for mechanical cooling or at least significantly reduce the size and cost of the equipment.

2. HEAT TRANSFER FUNDAMENTALS OF PASSIVE THERMAL CONTROL

2.1 Modes of heat transfer

In general, heat transfer is an energy transfer across a system boundary due to a temperature differential, [8-10]. Conceptually we deal with three basic forms:

- conduction
- convection
- and radiation.

Conduction represents an energy transfer across a system boundary due to a temperature difference by the mechanism of inter-molecular interactions. Conduction can occur through a solid, liquid or gas but, because the interactions occur at the molecular level, the material should appear stationary to the eye.

One dimensional conduction is described by the Fourier - Biot law:

$$\dot{Q}_x = -k \cdot A \cdot \frac{dT}{dx} \quad (1)$$

where k is defined as the thermal conductivity of the material, A is the area through which heat flows and dT/dx is the temperature gradient. For rectangular geometries, involving finite dimensions, this equation may be shown to reduce to the form (Fig. 1):

$$\dot{Q}_x = -k \cdot A \cdot \frac{\Delta T}{L} \quad (2)$$

where L is the distance measured between the two temperatures.

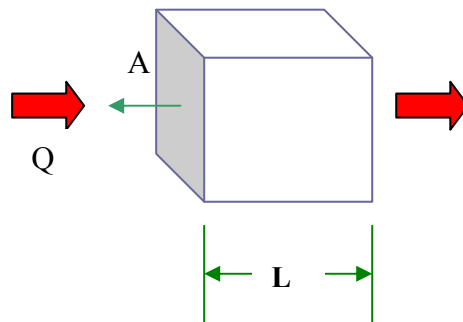


Figure 1. One dimensional conduction for rectangular geometries

In circular coordinates it may be convenient to work in the radial direction:

$$Q_r = -k \cdot A_r \cdot \frac{dT}{dr} \quad (3)$$

As noted previously, thermal conductivity is a thermodynamic property of a material. Recall the State Postulate from thermodynamics. For gases we find that thermodynamic properties of pure substances are functions of two independent thermodynamic intensive properties, say temperature and pressure. Thermal conductivity of real gases is largely independent of pressure and may be considered a function of temperature alone. For solids and liquids, properties are largely independent of pressure and depend on temperature alone, $k = k(T)$.

The thermal conductivity of most materials is a well behaved property, changing only slowly with temperature. Exceptions naturally occur between states where the material changes phase or crystalline structure. Generally the thermal conductivity of a solid or liquid will decrease with increasing temperature; gaseous materials will generally exhibit an increase in thermal conductivity with increasing temperature.

- Example: The inner and outer surfaces of a 5m x 6m brick wall of thickness 30cm and thermal conductivity 0.69W/m C are maintained at temperatures of 20 C and 5 C, respectively. Determine the rate of heat transfer through the wall in W.

Solution: From the Fourier-Biot law for one dimensional Cartesian conduction:

$$\dot{Q}_x = -k \cdot A \frac{\Delta T}{L}.$$

Substituting values:

$$Q = -(0.69 \text{ W/m}^\circ\text{C})(5 \text{ m} \cdot 6 \text{ m}) \frac{(5^\circ\text{C} - 20^\circ\text{C})}{0.3 \text{ m}} = 1035 \text{ W}.$$

Common application of PTC in refrigeration and cooling [11] deals with insulation material design, allowed slowing down the flow of heat. For example, building insulation slows all three types of heat flow (i.e. conduction, convection and radiation), though its greatest impact is on conduction. Most insulation materials are lightweight fibrous or cellular materials that enclose air or gas pockets. Conduction occurs both through the gas and the solid material separating pockets of gas. Convection occurs as air circulates through the insulation material, and is usually a minor component. Radiation occurs across air pockets in insulation or through air gaps in the building shell.

Convection is an energy transfer across a system boundary due to a temperature difference by the combined mechanisms of intermolecular interactions and bulk transport. Convection can occur only at a solid to fluid interface. Because bulk transport is involved motion of the fluid should be apparent to the eye.

Convection is described by Newton's law of cooling:

$$\dot{Q} = h \cdot A (T_{\text{surface}} - T_{\text{fluid}}) \quad (4)$$

where h is defined as the convection coefficient, A is the surface area being cooled, T_{surface} is the solid surface temperature and T_{fluid} is the bulk temperature of the cooling fluid.

Convection coefficients are experimentally derived parameters for a specific set of cooling conditions; they are not thermodynamic properties. Experimental correlations have been developed to describe convection coefficients for literally thousands of different geometries and cooling conditions.

- **Example:** For heat transfer purposes, a standing man can be modeled as a 30 cm diameter, 170 cm long vertical cylinder with both the top and bottom surfaces insulated and with the side surface at an average temperature of 34C. For a convection heat transfer coefficient of 15 W/m²C, determine the rate of heat loss from this man by convection in an environment at 20 C.

Solution: From Newton's law of cooling:

$$\dot{Q} = \left(15 \frac{\text{W}}{\text{m}^2 \text{ }^\circ\text{C}} \right) [(\pi \cdot 0.3 \text{ m}) \cdot 1.7 \text{ m}] \cdot (34 \text{ }^\circ\text{C} - 20 \text{ }^\circ\text{C}) = 336 \text{ W}$$

Radiation is an energy transfer across a system boundary due to a temperature difference by the mechanism of photon emission or electromagnetic waves. Because radiation does not involve molecular interactions it can occur across a vacuum.

Radiation is described by the Stefan-Boltzman equation:

$$\dot{E}_{\text{max}} = \sigma \cdot A \cdot T_{\text{abs}}^4 \quad (5)$$

where E is the maximum theoretical rate of energy emission, σ is the Stefan-Boltzman constant and has the value $5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$, A is the surface area from which emission occurs, T_{abs} is the absolute temperature of the emitting surface.

Real surfaces do not emit radiation at the maximum, theoretical rate but at some lower fraction. The thermal emissivity, ε , is defined as the fraction of energy emitted by a real surface rationed to that of an ideal surface. Then

$$\dot{E}_{\text{max}} = \varepsilon \cdot \sigma \cdot A \cdot T_{\text{abs}}^4 \quad (6)$$

Closely related to the emissivity, ε , of a surface are the absorptivity, α , which is defined as the fraction of radiant energy incident upon a surface which is absorbed, the reflectivity, ρ , the fraction of incident energy reflected at a surface, and the transmissivity, τ , the fraction of incident energy transmitted through a surface, (Notice, from the Conservation of Energy $\alpha + \rho + \tau = 1$.) see Fig. 2.

Radiation can be a complex process involving many surfaces. A simple case involves radiation between a single surface, with uniform properties and at a uniform surface temperature, radiating to a large enclosure, also at a uniform temperature:

$$\dot{Q} = \varepsilon \cdot \sigma \cdot A \cdot (T_{abs}^4 - T_{enclosure}^4) \quad (7)$$

The net energy transferred between bodies, described by the equation above, may also be written as the difference in emission between the two bodies:

$$\dot{Q} = \varepsilon \cdot A \cdot \Delta E \quad (8)$$

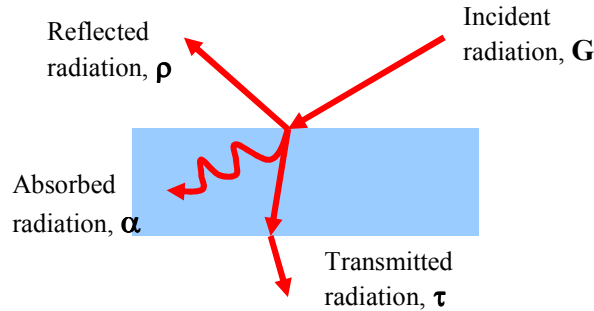


Figure 2. Distribution of incident energy

- **Example:** The outer surface of a spacecraft in space has an emissivity of 0.8 and an absorptivity of 0.3 for solar radiation. If solar radiation is incident on the spacecraft at a rate of 1000 W/m^2 , determine the surface temperature of the spacecraft when the radiation emitted equals the solar energy absorbed.

Solution: Determine absorbed energy and emitted radiation:

$$E_{abs} = \alpha E_{incident} = 0.3 (1000 \text{ W/m}^2) \cdot A = 300 A,$$

$$E_{emitted} = \varepsilon \sigma A T^4 = 0.8 \cdot (5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4) \cdot A \cdot T^4 (\text{K}^4).$$

Equating the energy absorbed and emitted:

$$0.3 \cdot 1000 \text{ W/m}^2 \cdot A = 0.8 \cdot 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \cdot A \cdot T^4.$$

Rearrange, solving for T

$$T = \{(300 A) / [0.8(5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4) \cdot A]\}^{0.25} = 285 \text{ K}$$

2.2 Composite systems

Consider the two blocks, A and B, as shown in Fig. 3a. They are insulated on top, bottom, front and back. Assume that $T_1 > T_2$ so that the direction of heat flow is from the left to the right. Since the energy will flow first through block A and then through block B, we say that these blocks are thermally in a series arrangement.

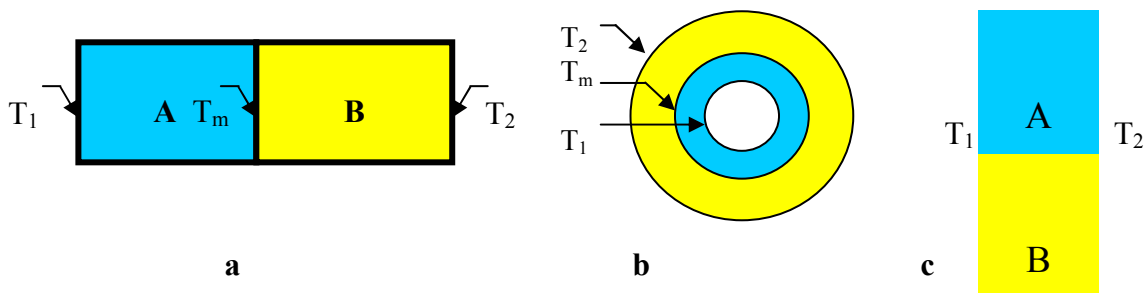


Figure 3. Composite systems: a, b – in series arrangement; c – in parallel arrangement

A similar situation exists with the two cylinders shown in Fig. 3b. Assume that $T_1 > T_2$ so that the direction of heat flow is outward radially.

Consequently, the composite resistance of two simple elements placed in series is the sum of the individual element resistances

$$R_{composite} = R_A + R_B \quad (9)$$

We see from the 1st law of Thermodynamics that the heat flow must be equal through both elements A and B $q_A = q_B = q$.

Then:

$$q \cdot (R_A + R_B) = T_1 - T_2 = q \cdot R_{composite} = \Delta T \quad (10)$$

Consider the two blocks, A and B, as shown in Fig. 3c. They are insulated on top, bottom, front and back. Assume that $T_1 > T_2$ so that the direction of heat flow is from the left to the right. Since the energy flow will divide, part going through element A and part through B, the elements are said to be thermally in parallel. The reciprocal composite resistance of two simple elements placed in parallel is the sum of the reciprocal individual element resistances

$$1/R_{composite} = 1/R_A + 1/R_B \quad (11)$$

23 Cooling fins

The cooling fin is an important device, commonly found in systems where air is used as a coolant. In this lecture consideration will be limited to rectangular fins of constant area. Annular fins or fins involving a tapered cross section may be analyzed by similar methods, but will require a degree of preparation to handle the more complicated equations which result, [12]. Numerical methods of integration or computer programs such as *MATCAD* or *MAPLE* can be used to advantage in such cases.

Consider the cooling fin shown in Fig. 4.

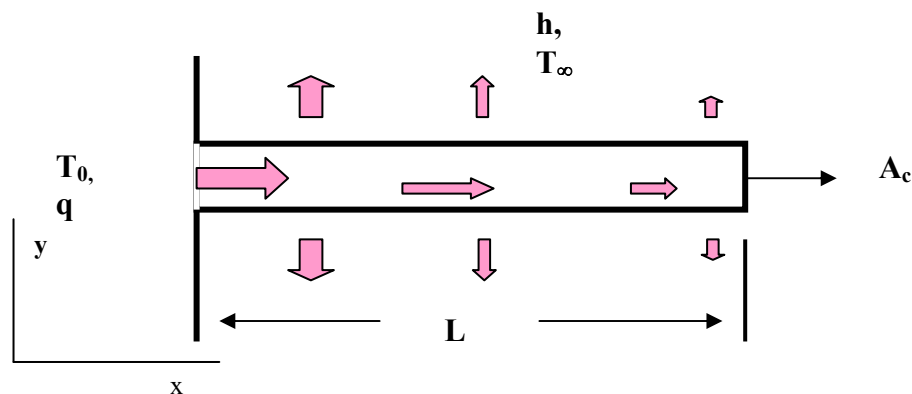


Figure 4. Design scheme of a cooling fin

The fin is situated on the surface of a hot surface at T_0 and surrounded by a coolant at temperature T_∞ , which cools with convective coefficient, h . The fin has a cross sectional area, A_c , (this is the area through which heat is conducted) and an overall length, L .

Note that as energy is conducted down the length of the fin, some portion is lost, by convection, from the sides. Thus the heat flow varies along the length of the fin. The equivalent system will involve the introduction of heat sinks (negative heat sources), which remove an amount of energy equivalent to what would be lost through the sides by convection.

From the general conduction equation as simplified for one-dimension, steady state conduction with sources we can obtain the fin equation:

$$\frac{d^2T}{dx^2} - m^2 \cdot (T - T_\infty) = 0 \quad (12)$$

where $m^2 = hP/(kA_c)$, P is the perimeter around the fin.

This is the second order differential equation that we will solve for each fin analysis. Prior to solving, a couple of simplifications should be noted. First, we see that h , P , k and A_c are all independent of x in the defined system (They may not be constant if a more general analysis is desired.).

Next we notice that the equation is non-homogeneous (due to the T_∞ term). Recall that non-homogeneous differential equations require both a general and a particular solution. We can simplify this equation by introducing the temperature relative to the surroundings $\theta \equiv T - T_\infty$.

The fin equation has two solutions:

$$\theta = A \cdot e^{mx} + B \cdot e^{-mx} \quad \text{and} \quad \theta = C \cdot \cosh(mx) + D \cdot \sinh(mx) \quad (13)$$

where A , B , C and D are arbitrary constants to provide for an arbitrary linear combination of the independent solutions. The constants also serve as constants of integration. Generally the exponential solution is used for very long fins, the hyperbolic solutions for other cases.

Since the solution results in 2 constants of integration we require 2 boundary conditions. One end of the fin will be attached to a hot surface and will come into thermal equilibrium with that surface. For very long fins, the end located a long distance from the heat source will approach the temperature of the surroundings:

$$\theta(0) = T_0 - T_\infty \equiv \theta_0 \quad \text{and} \quad \theta(\infty) = 0 \quad (14)$$

The general temperature profile for a very long fin is then:

$$\theta(x) = \theta_0 \cdot e^{-mx} \quad (15)$$

If we wish to find the heat flow through the fin, we may apply Fourier-Biot Law and differentiate the temperature profile:

$$q = k \cdot A_c \cdot \theta_0 \left(\frac{h \cdot P}{k \cdot A_c} \right)^{0.5} \cdot \exp(-mx) = \sqrt{h \cdot P \cdot k \cdot A_c} \exp(-mx) \cdot \theta_0 \quad (16)$$

Often we wish to know the total heat flow through the fin, i.e. the heat flow entering at the base ($x=0$)

$$q = \sqrt{h \cdot P \cdot k \cdot A_c} \cdot \theta_0 \quad (17)$$

Finite fins are treated similarly, but the mathematics is simplified if (a) the coordinate system is arranged to start from the tip, rather than the base, of the fin and (b) the hyperbolic form of the solution is used.

The fin efficiency is defined as the ratio of the energy transferred through a real fin to that transferred through an ideal fin. An ideal fin is thought to be one made of a perfect or infinite conductor material. A perfect conductor has an infinite thermal conductivity so that the entire fin is at the base material temperature.

The heat transfer through any fin can now be written as:

$$q \cdot [\eta \cdot h \cdot A_c]^{-1} = T - T_\infty \quad (18)$$

where $\eta = \sqrt{\frac{k \cdot A_c}{h \cdot P}} \frac{\theta_L \cdot \tanh(m \cdot L)}{L \cdot \theta_0} = \frac{\tanh(m \cdot L)}{m \cdot L}$ is the fin efficiency.

2.4 Some issues of free convection theory

Consider the process of convective cooling, as we pass a cool fluid past a heated wall. Near any wall a fluid is subject to the no slip condition; that is, there is a stagnant sub layer. Since there is no fluid motion in this layer, heat transfer is by conduction in this region. Above the sub layer is a region where viscous forces retard fluid motion; in this region some convection may occur, but con-

duction may well predominate. A careful analysis of this region allows us to use our conductive analysis in analyzing heat transfer. This is the basis of the convective theory.

The fundamental definition declares that in free convection fluid motion is due to buoyancy forces within the fluid (Fig. 5), while in forced convection it is externally imposed. This is valid in a technical sense, but hardly serves to help the engineer in determining the appropriate convective correlation to apply. In practice, this definition has been found to be inadequate. In particular, in a natural convection boiler while the circulation is due to free convection, the convective heat transfer is by forced convection.

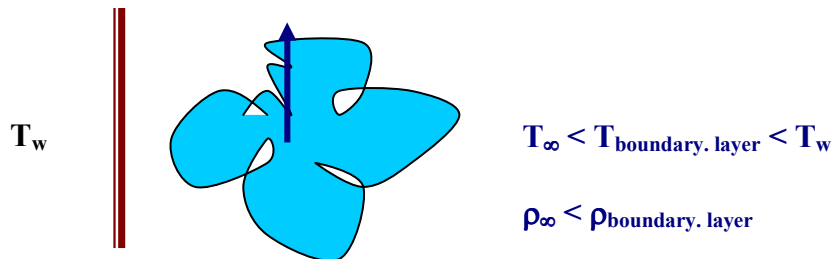


Figure 5. To the definition of free convection

Saturated liquid enters the boiler wall tubes, where the water is heated by hot combustion gases inside the boiler. A portion of the water vaporizes and the density of the mixture is decreased, causing the water/steam column to rise. In the steam drum, the water and steam separate. The dry steam goes to the superheater and the water is allowed to flow back to the mud drum. The return line is outside the boiler so that the fluid remains liquid. Because of the density difference between the two legs, a natural circulation pattern forms.

Compare the velocity profiles for forced and natural convection shown in Fig. 6. In the case of the boiler wall tubes, the fluid inside the tubes flows upward with a velocity profile that extends across the entire cross section of the pipe. This profile looks like the one for forced convection, producing a boundary layer that also looks like forced convection. Because the boundary layer profile does not duplicate that found in natural, or free, convection heat transfer, forced convection correlations apply. If the velocity increases as the wall is approached, then natural convection is present.

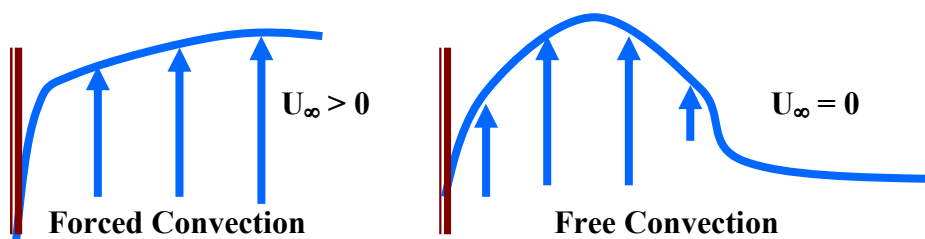


Figure 6. Velocity profiles for forced and natural convection

The thermodynamic property which describes the change in density leading to buoyancy in the Coefficient of Volumetric Expansion, β :

$$\beta \equiv -\frac{1}{\rho} \cdot \frac{\partial \rho}{\partial T} \Bigg|_{P=Const} \quad (19)$$

Because U_∞ is always zero, the Reynolds number, $[\rho \cdot U_\infty \cdot D] / \mu$, is also zero and is not suitable to describe the flow in the system. Instead, we introduce a new parameter for natural convection, the

$$\text{Grashof Number: } Gr = \frac{\rho^2 \cdot g \cdot \beta \cdot \Delta T \cdot L^3}{\mu^2} = \frac{\left(\frac{\rho \cdot g \cdot \beta \cdot \Delta T \cdot L^3}{L^2} \right) \cdot (\rho \cdot U_{\max}^2)}{\mu^2 \cdot \frac{U_{\max}^2}{L^2}} = \frac{\text{Buoyant Force}}{\text{Viscous Force}} \quad (20)$$

Generally the characteristic length used in the correlation relates to the distance over which the boundary layer is allowed to grow. In the case of a vertical flat plate this will be x or L , in the case of a vertical cylinder this will also be x or L ; in the case of a horizontal cylinder, the length will be d , see Fig. 7.

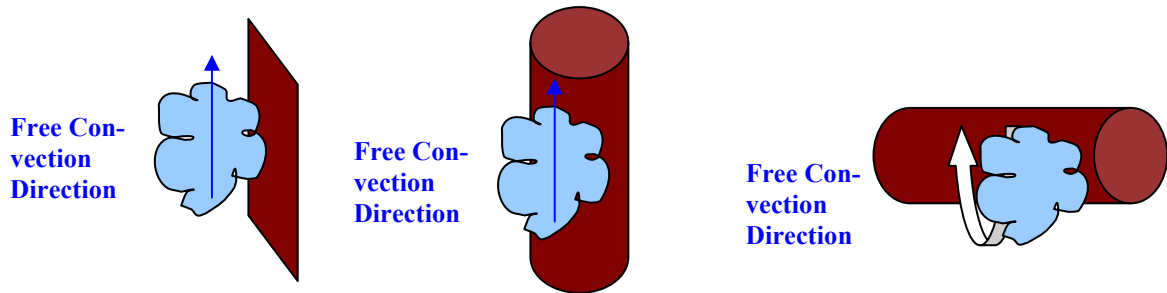


Figure 7. To the definition of characteristic length

Consider the flow between two surfaces, each at different temperatures. Under developed flow conditions, the interstitial fluid will reach a temperature between the temperatures of the two surfaces and will develop free convection flow patterns. The fluid will be heated by one surface, resulting in an upward buoyant flow, and will be cooled by the other, resulting in a downward flow. If the surfaces are placed closer together, the flow patterns will begin to interfere as it is shown in Fig. 8.

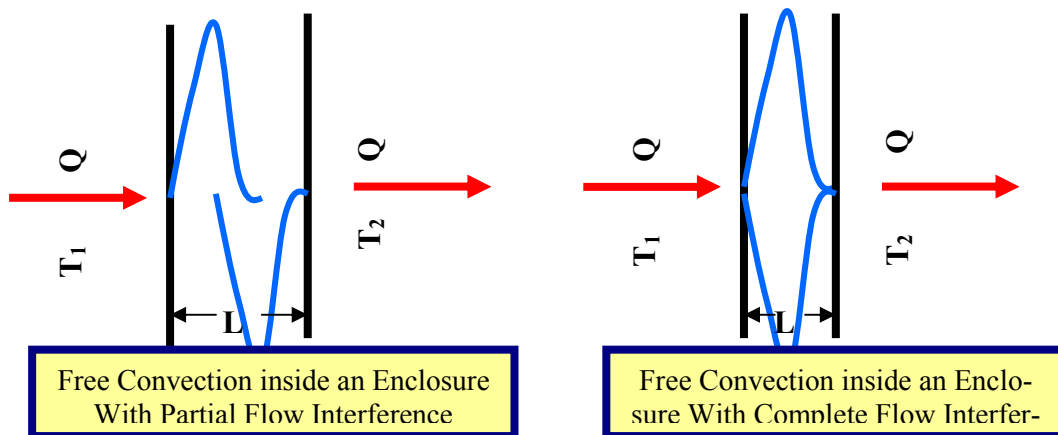


Figure 8. Free convection inside an enclosure

The transition in enclosures from convection heat transfer to conduction heat transfer occurs at what is termed the *Critical Rayleigh Number*. Note that this terminology is in clear contrast to forced convection where the critical Reynolds number refers to the transition from laminar to turbulent flow.

$$Ra_{crit} = 1000 \quad (\text{Enclosures with horizontal heat flow})$$

$$Ra_{crit} = 1728 \quad (\text{Enclosures with vertical heat flow})$$

2.5 Several problems of radiation theory

We are well acquainted with a wide range of electromagnetic phenomena in modern life. These phenomena are sometimes thought of as wave phenomena and are, consequently, often described in terms of electromagnetic wave length, λ . Examples are given in terms of the wave distribution shown in Fig. 9.

The Stefan-Boltzman equation (5) provides a method of determining the total energy leaving a surface, but gives no indication of the direction in which it travels. As we continue our study, we will want to be able to calculate how heat is distributed among various objects, [13]. For this purpose, we will introduce the radiation intensity, defined as the energy emitted from an ideal body, per unit projected area, per unit time, per unit solid angle:

$$I = \frac{dq}{\cos \theta \cdot dA_1 \cdot d\Omega} \quad (2)$$

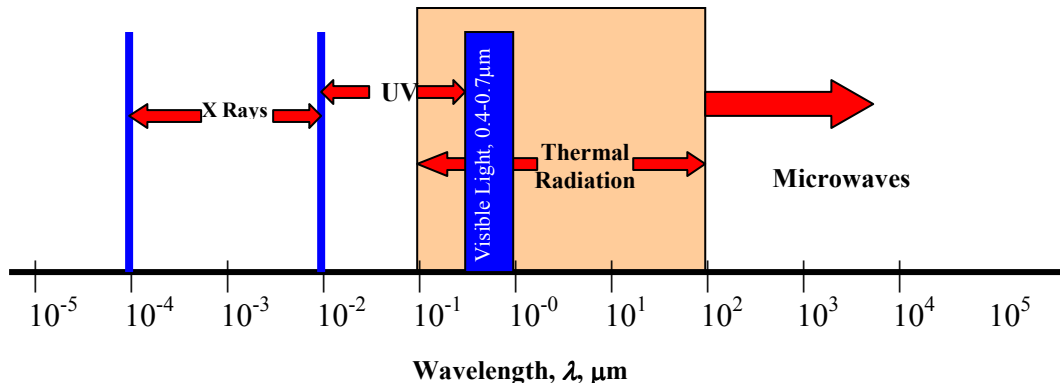


Figure 9. Electromagnetic phenomena

Consider two flat, infinite planes, surface A and surface B, both emitting radiation toward one another. Surface B is assumed to be an ideal emitter, i.e. $\epsilon_B = 1.0$. Surface A will emit radiation according to the Stefan-Boltzman law as:

$$E_A = \epsilon_A \cdot \sigma \cdot T_A^4 \quad (2)$$

and will receive radiation as:

$$G_A = \alpha_A \cdot \sigma \cdot T_B^4 \quad (2)$$

The net heat flow from surface A will be:

$$q = \epsilon_A \cdot \sigma \cdot T_A^4 - \alpha_A \cdot \sigma \cdot T_B^4 \quad (24)$$

Now suppose that the two surfaces are at exactly the same temperature. The heat flow must be zero according to the 2nd law. It follows then that $\alpha_A = \epsilon_A$.

Because of this close relation between emissivity, ϵ , and absorptivity, α , only one property is normally measured and this value may be used alternatively for either property. Let's not lose sight of the fact that, as thermodynamic properties of the material, α and ϵ may depend on temperature. In general, this will be the case as radiative properties will depend on wavelength, λ . The wavelength of radiation will, in turn, depend on the temperature of the source of radiation.

In the design of solar collectors, engineers have long sought a material which would absorb all solar radiation, ($\alpha = 1$, $T_{\text{sun}} \sim 5600\text{K}$) but would not re-radiate energy as it came to temperature ($\epsilon \ll 1$, $T_{\text{collector}} \sim 400\text{K}$). NASA developed anodized chrome, commonly called "black chrome" as a result of this research.

Within the visual band of radiation, any material, which absorbs all visible light, appears as black. Extending this concept to the much broader thermal band, we speak of surfaces with $\alpha = 1$ as also

being “black” or “thermally black”. It follows that for such a surface, $\varepsilon = 1$ and the surface will behave as an ideal emitter. The terms ideal surface and black surface are used interchangeably.

Rearranging the Radiation intensity equation to express the heat radiated:

$$dq = I \cdot \cos \theta \cdot dA_1 \cdot d\Omega \quad (25)$$

Next we will project the receiving surface onto the hemisphere surrounding the source. First find the projected area of surface dA_2 , $dA_2 \cdot \cos \theta_2$. (θ_2 is the angle between the normal to surface 2 and the position vector, R), see Fig. 10. Then find the solid angle, Ω , which encompasses this area.

To obtain the entire heat transferred from a finite area, dA_1 , to a finite area, dA_2 , we integrate over both surfaces:

$$q_{1 \rightarrow 2} = \int_{A_2} \int_{A_1} \frac{I \cdot \cos \theta_1 \cdot dA_1 \cdot \cos \theta_2 dA_2}{R^2} \quad (26)$$

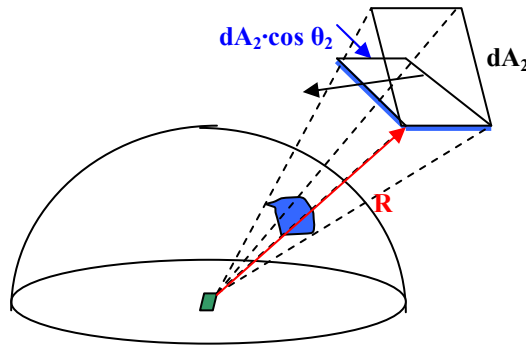


Figure 10 To the definition of view factor

To express the total energy emitted from surface 1, we recall the relation between emissive power, E , and intensity, I :

$$q_{emitted} = E_1 \cdot A_1 = \pi \cdot I \cdot A_1 \quad (27)$$

Define the view factor, F_{1-2} , as the fraction of energy emitted from surface 1, which directly strikes surface 2:

$$F_{1 \rightarrow 2} = \frac{q_{1 \rightarrow 2}}{q_{emitted}} = \frac{\int_{A_2} \int_{A_1} \frac{I \cdot \cos \theta_1 \cdot dA_1 \cdot \cos \theta_2 dA_2}{R^2}}{\pi \cdot I \cdot A_1} \quad (28)$$

after algebraic simplification this becomes:

$$F_{1 \rightarrow 2} = \frac{1}{A_1} \cdot \int_{A_2} \int_{A_1} \frac{\cos \theta_1 \cdot \cos \theta_2 \cdot dA_1 \cdot dA_2}{\pi \cdot R^2} \quad (29)$$

The determination of the view factor is difficult, but view factors for different materials are available in literature.

In order that we might apply conservation of energy to the radiation process, we must account for all energy leaving a surface. We imagine that the surrounding surfaces act as an enclosure about the heat source which receives all emitted energy. Should there be an opening in this enclosure through which energy might be lost, we place an imaginary surface across this opening to intercept this portion of the emitted energy. For an N surfaced enclosure, we can obtain the Conservation rule

$$\sum_{j=1}^N F_{i,j} = 1 \quad (30)$$

We may write the view factor from surface i to surface j as:

$$A_i \cdot F_{i \rightarrow j} = \int_{A_j} \int_{A_i} \frac{\cos \theta_i \cdot \cos \theta_j \cdot dA_i \cdot dA_j}{\pi \cdot R^2} \quad (31)$$

Similarly, between surfaces j and i :

$$A_j \cdot F_{j \rightarrow i} = \int_{A_j} \int_{A_i} \frac{\cos \theta_j \cdot \cos \theta_i \cdot dA_j \cdot dA_i}{\pi \cdot R^2} \quad (32)$$

Here we obtained the Reciprocity relationship based on the identity of the integrals:

$$A_i \cdot F_{i \rightarrow j} = A_j \cdot F_{j \rightarrow i} \quad (33)$$

- Example: Consider two concentric spheres shown in the Fig. 11. All radiation leaving the outside of surface 1 will strike surface 2. Part of the radiant energy leaving the inside surface of object 2 will strike surface 1, part will return to surface 2. To find the fraction of energy leaving surface 2 that strikes surface 1, we apply reciprocity:

$$A_2 \cdot F_{2,1} = A_1 \cdot F_{1,2} \Rightarrow F_{2,1} = \frac{A_1}{A_2} \cdot F_{1,2} = \frac{A_1}{A_2} = \left[\frac{D_1}{D_2} \right]^2$$

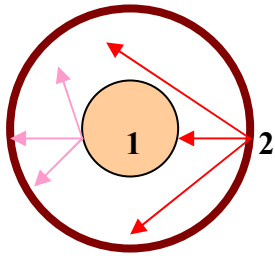


Figure 11 To the definition of the Reciprocity

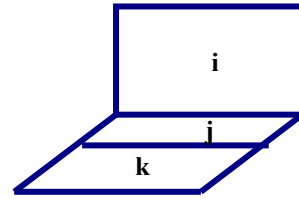


Figure 12 To the definition of the Associative rule

Consider the set of surfaces shown in the Fig. 12. Clearly, from conservation of energy, the fraction of energy leaving surface i and striking the combined surface $j+k$ will equal the fraction of energy emitted from i and striking j plus the fraction leaving surface i and striking k that is known as the Associative rule:

$$F_{i \rightarrow (j+k)} = F_{i \rightarrow j} + F_{i \rightarrow k} \quad (34)$$

Radiosity, J , is defined as the total energy leaving a surface per unit area and per unit time.

$$J = \varepsilon \cdot E_b + \rho \cdot G \quad (35)$$

This may initially sound much like the definition of emissive power, but the Fig. 13 will help to clarify the concept.

Consider the two surfaces shown in the Fig. 14. Radiation will travel from surface i to surface j and will also travel from j to i .

Reciprocity relationship yields

$$q_{j \rightarrow i (net)} = J_i \cdot A_i \cdot F_{i \rightarrow j} - J_j \cdot A_j \cdot F_{j \rightarrow i} = A_i \cdot F_{i \rightarrow j} \cdot (J_i - J_j) \quad (36)$$

The net energy leaving a surface will be the difference between the energy leaving a surface and the energy received by a surface:

$$q_{i \rightarrow} = [\varepsilon \cdot E_b - \alpha \cdot G] \cdot A_i \quad (37)$$

Combine eq. (37) with the definition of radiosity to eliminate G . Then, assume opaque surfaces so that $\alpha + \rho = 1$ $\rho = 1 - \alpha$, substitute for ρ and put the resulting equation over a common denominator:

$$q_{i \rightarrow} = \left[\frac{(1 - \alpha) \cdot \varepsilon \cdot E_b - \alpha \cdot J + \alpha \cdot \varepsilon \cdot E_b}{1 - \alpha} \right] \cdot A_i = \left[\frac{\varepsilon \cdot E_b - \alpha \cdot J}{1 - \alpha} \right] \cdot A_i \quad (38)$$

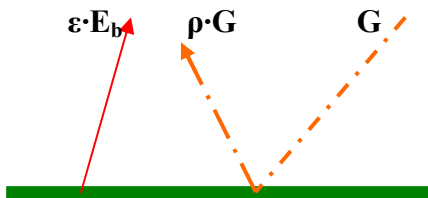


Figure 13. To the definition of the Radiosity

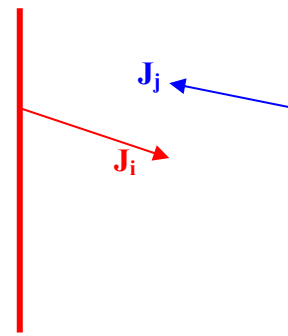


Figure 14. To the definition of the net energy

A radiator is a device with a large surface area used to radiate heat. For an active system, radiators consist of a lattice work of fluid loops (usually freon). Radiator size depends on both heat loads and required temperature. This is the primary system of heat rejection on manned space missions. More advanced radiator concepts include droplet radiators [14] and bubble membrane radiators [15]. Space radiator is a heat dissipater located on the outer surface of a spacecraft and used to radiate thermal energy to deep space. Second surface mirrors are considered space radiators.

There are three types of external thermal radiation which can affect a spacecraft: solar flux, albedo and radiation emitted by the Earth. Solar flux is the heat flux emanating from the Sun. For an Earth satellite the average solar flux is 1353 W/m^2 . This is the average flux at one astronomical unit. For an interplanetary vessel, the solar flux will decrease as the craft moves away from the Sun. This varies as the inverse square of the distance from the sun.

For a spacecraft with solar panels, antennas, etc., the projected area can be calculated by shining a light from a distance and determining which surfaces are illuminated, or by taking a photograph of the craft.

The albedo flux is the solar heat reflected by the Earth's surface and the atmosphere. This type of flux should be considered by any spacecraft in an orbit around a planet. For example, imagine a spacecraft placed in low Earth orbit, and then fired on a trajectory to another planet where it will also be injected into a low orbit. The spacecraft would be exposed to the albedo of the target planet, but it would be unaffected by albedo during the trajectory from Earth to the planet.

For Earth, the recommended average value for the albedo coefficient is 0.30 ± 0.02 , but the value varies from 0.1 to 0.8. This is just an approximation since the albedo flux varies according to cloud cover, ice formations, orientation of the spacecraft, orientation of the sun, etc.

Celestial bodies absorb solar heat and emit it in the form of thermal radiation. When a spacecraft is near the Earth or near a target planet, it experiences heating as a result of thermal radiation. The thermal radiation is emitted according to the Stefan-Boltzman Law.

It should be emphasized that the conduction and radiation analogies should not be mixed. When both mechanisms are present, they should be analyzed in separate circuits.

- Example: The American Indians of the south-west were able to produce ice in the desert areas. To do this, they would place a container of water in a quiescent location under the night sky. Space, in low earth orbit, remains at a temperature of about 243 K. In the cloudless, clear skies of the southwest, the water will have a direct view of space. Below the bowl, a small animal pelt is placed to insulate the bowl from the ground. Find the maximum air temperature at which water in the bowl will freeze, Fig. 15.

Solution: The bowl of water may be seen as a small object placed in a large enclosure. Since space, represented by area 2, will be large the surface resistance for space will be zero.

The rate at which energy is radiated from the water surface into space is

$$q_{rad} = \epsilon_w \cdot A_w \cdot \sigma \cdot (T_w^4 - T_\infty^4)$$

The rate at which heat is convected to the water from atmospheric air is equal to

$$q_{conv} = h \cdot A_w \cdot (T_{air} - T_w)$$

The system will reach equilibrium when the heat gained by convection is exactly equal to that lost by radiation:

$$q_{rad} = q_{conv} \text{ and } h \cdot A_w \cdot (T_{air} - T_w) = \varepsilon_w \cdot A_w \cdot \sigma \cdot (T_w^4 - T_\infty^4).$$

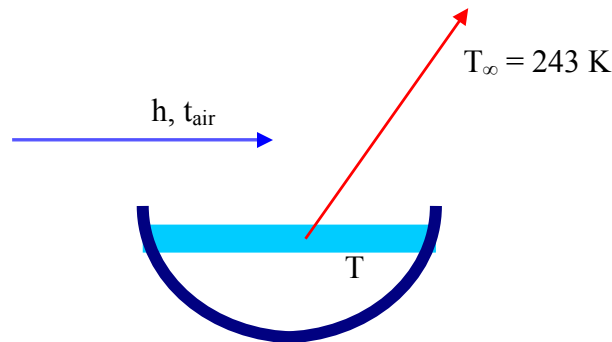


Figure 15. Combined mechanism of free convection and radiation

The area of the water, A_w , cancels. From [9], we find that $\varepsilon_w = 0.96$ and $h \approx 7 \text{ W/m}^2 \text{ K}$. Substituting these values:

$$7 \text{ W/m}^2 \cdot \text{K} \cdot (T_{air} - 273 \text{ K}) = \varepsilon_w \cdot 5.6710^{-8} \text{ W/m}^2 \cdot \text{K}^4 \cdot [(273 \text{ K})^4 - (243 \text{ K})^4].$$

Solving for T_{air}

$$T_{air} = 289 \text{ K}.$$

It is a surprisingly high temperature for which ice to form.

3. PASSIVE THERMAL CONTROL SYSTEMS

3.1 Passive thermal control components

Thermal control coatings surfaces, such as black and white paints, and gold, silver, and aluminum foils have special radiation properties. Coatings may be combined to obtain a more desirable average value for emissivity and absorptivity. In general, it is desirable to have a high value of emissivity and a low value of absorptivity in order to maximize the heat rejection into space and to minimize the solar input. Thermal coatings are very efficient and lightweight. Unfortunately, they will degrade over time.

Multi layer insulation reduces the rate of heat flow per unit area between two boundary surfaces and prevents a large heat influx. Sensors and payloads can be wrapped in insulation blankets to thermally isolate them and reduce thermal control requirements.

Thermal doublers are heat sinks made of a highly conductive material placed in thermal contact with a component. Heat is conducted to the sink during an increase in temperature and then dispersed by radiation or conduction. The process also works in reverse and keeps components from experiencing severe cooling. They can also be used to spread heat out over radiator surfaces, and are frequently used to control the temperature of electrical equipment that has high dissipation of cyclical variation in power dissipation.

Phase change devices absorb thermal energy by changing from a solid to a liquid. As the temperature decreases, the material re-solidifies. It is especially useful for electrical equipment that experiences short power spikes. The main disadvantage of phase change devices is that they are unable to absorb any more heat after melting, which allows the temperature to increase. A common type of phase change device is some type of wax in an aluminum container. These devices can be used between a cold plate and the primary heat dissipation device.

Cold plates (Fig. 16) are used for mounting heat dissipating equipment. In an active system, there are fluid passages within the plate itself. The fluid is then pumped to a radiator. For a passive system, the cold plate is usually combined with the radiator.

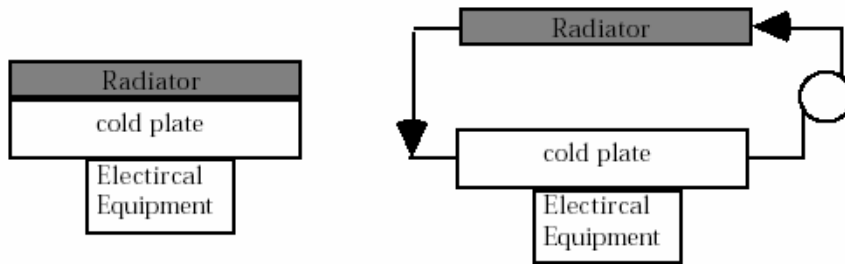


Figure 16. Passive and active cold plate systems

32 Numerical example on spacecraft thermal control system design

The steps described below follow the flowchart shown in Figure 17. These steps are intended to be used only as a guide, [16-22]. In addition, this example assumes that the spacecraft will always be between the Sun and the Earth. Therefore, there will be no thermal radiation incident upon the spacecraft from earth.

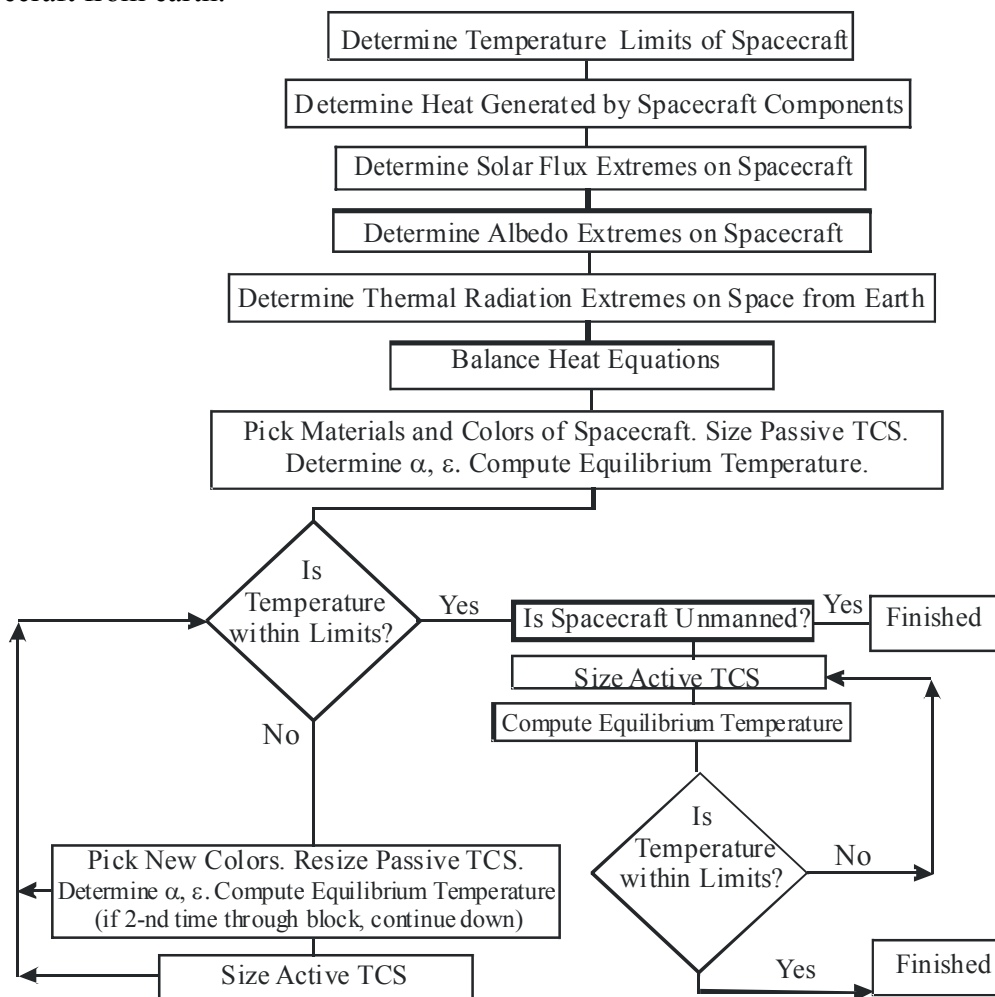


Figure 17. Top down design proces for a Thermal Control System

1) Determine temperature limits of spacecraft components.

Consider an isothermal spy satellite with one camera in a LEO. The satellites projected area is 1m^2 , and it is in a circular orbit. Table 1 lists several components and their associated temperature ranges. However, for this example, assume that the spy camera operates at a temperature of 320K.

Table 1
Thermal Design Temperature Limit ($^{\circ}\text{C}$) Min/Max

Subsystem/Equipment	Non-operating/ Turn-on	Operating
Communications		
Receiver	-30/+55	+10/+45
Input multiplex	-30/+55	-10/+30
Output multiplex	-30/+55	-10/+40
TWTA	-30/+55	-10/+55
Antenna	-170/+90	-170/+90
Electric power		
Solar array wing	160/+80	-160/+80
Battery	-10/+25	0/+25
Shunt assembly	-45/+65	-45/+65
Attitude control		
Earth/Sun, sensor	-30/+55	-30/+50
Angular rate assembly	-30/+55	+1/+55
Momentum wheel	-15/+55	+1/+45
Propulsion		
Solid apogee, motor	+5/+35	---
Propellant tank	+10/+50	+10/+50
Thruster catalyst bed	+10/+120	+10/+120
Structure		
Pyrotechnic mechanism	-170/+55	-115/+55
Separation clamp	-40/+40	-15/+40

2) Determine heat generated by spacecraft components. The manufacturer of the camera states that this camera generates 100 W of heat. Determine solar flux extremes on spacecraft. When the spy satellite is in the earth's shadow, the solar flux will be zero. Otherwise, the recommended value for the solar flux at LEO is $S = 1353\text{ W/m}^2$. The ratio of the projected area of the spacecraft is 0.78 due to the shield. Therefore, the solar heat rate, Q_{sun} , is 1055 W.

3) Determine albedo extremes on spacecraft. The recommended average value for the albedo coefficient of the Earth is given as $a = 0.3 \pm 0.02$. Thus, the albedo flux constant can be calculated by $\Phi = S_a = 405.9\text{ W/m}^2$. The total heat rate is given as:

$$Q_{al} = S_a A = 405.9 \times 1 = 405.9\text{ W}.$$

4) Balance heat equation $Q_{store} = Q_{in} - Q_{eq} + Q_{diss}$. Since our satellite is isothermal, Q_{store} is zero. $Q_{in} = 1461.2\text{ W}$ and $Q_{eq} = 100\text{ W}$. Therefore, $Q_{diss} = -1561.2\text{ W}$.

5) Pick Materials and colors of spacecraft (determine α , ϵ , τ). Size passive TCS. Compute equilibrium temperature.

Suppose we choose a material and a coating which gives an ϵ of 0.8, and we know that the total surface area of the satellite exposed to the Sun is $A = 1\text{ m}^2$. Since $Q_{diss} = \epsilon A \sigma T^4$, we can solve for the equilibrium temperature, T .

$$T = \left[\frac{Q_{diss}}{A \epsilon \sigma} \right]^{0.25} = \left[\frac{1561.2}{1 \cdot 0.8 \sigma} \right]^{0.25} = 430\text{K}$$

Table 2 lists several materials and coatings with their respective properties. Since the acceptable temperatures for our satellite is 320 K, we can see that the equilibrium temperature of the satellite is too high. Now passive radiators must be added in order to lower the equilibrium temperature. Determine the size of the radiator required to maintain the equilibrium temperature at 320 K

$$Q_{\text{prad}} = \varepsilon A \sigma \eta T^4 = 0.8 \times A \times 5.7 \times 10^{-8} \times 0.9 \times 320^4 = 1086W.$$

This gives a total surface area for the radiators of 1.99 m². The thermal control system design is now complete.

If this were a manned mission instead of a simple spy satellite, we would have had to add an active TCS to be able to fine tune the passive TCS and keep the astronauts safe.

Table 2
Properties of Materials and Coatings

Typical Surface	Application	Solar Absorptivity, α		Emissivity, ε	
		Beginning of life	End of life (7 years)	Beginning of life	End of life (7 years)
Paint	Interior structure	0.9	0.9	0.9	0.9
White paint	Antenna reflector	0.2	0.6	0.9	0.9
Optical solar reflector	North and south panel radiators	0.08	0.21	0.8	0.8
Graphite/ epoxy	Solar panel and antenna structure	0.84	0.84	0.85	0.85
Aluminized kapton	Thermal insulation	0.35	0.50	0.6	0.6
Tiodized titanium	Apogee motor thermal shield	0.6	0.6	0.6	0.6
Aluminum, aluminum tape insulation deposited aluminum	Propellant	0.12	0.18	0.06	0.06
Anodized aluminum	Interior structure	0.2	0.6	0.8	0.8
Solar cells	Solar panels	0.65 0.75	0.65 0.75	0.82	0.82
Gold		0.2 0.3		0.03 0.06	

4. PASSIVE COOLING STRATEGIES FOR BUILDING DESIGN

Passive cooling is based on the interaction of the building and its surroundings. Before adopting a passive cooling strategy, you must be sure that it matches your local climate.

In his book [24] Brown identifies four passive cooling strategies: natural ventilation, evaporative cooling, high thermal mass and high thermal mass with night ventilation. All these passive cooling strategies rely on daily changes in temperature and relative humidity.

A primary strategy for cooling buildings without mechanical assistance (passive cooling) in hot humid climates is to employ natural ventilation, which depends solely on air movement to cool occupants. Window openings on opposite sides of the building enhance cross ventilation driven by breezes. Since natural breezes can't be scheduled, designers often choose to enhance natural ventilation using tall spaces within buildings called stacks. With openings near the top of the stack, warm air can escape, while cooler air enters the building from openings near the ground. Ventilation requires the building to be open during the day to allow air flow.

Wing walls (Fig. 18) are vertical so lid panels placed alongside of windows perpendicular to the wall on the windward side of the house.

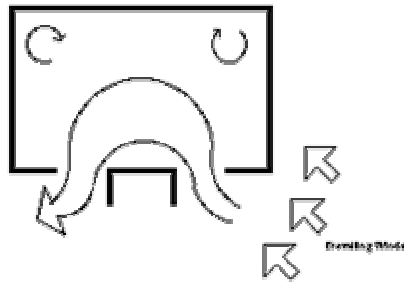


Figure 18. Top view of wing walls airflow pattern

Wing walls will accelerate the natural wind speed due to pressure differences created by the wing wall.

A thermal chimney employs convective currents to draw air out of a building. By creating a warm or hot zone with an exterior exhaust outlet, air can be drawn into the house ventilating the structure. Sunrooms can be designed to perform this function, see Fig. 19. The excessive heat generated in a south facing sunroom during the summer can be vented at the top. With the connecting lower vents to the living space open along with windows on the north side, air is drawn through the living space to be exhausted through the sunroom upper vents. (The upper vents from the sunroom to the living space and any side operable windows must be closed and the thermal mass wall in the sunroom must be shaded.)

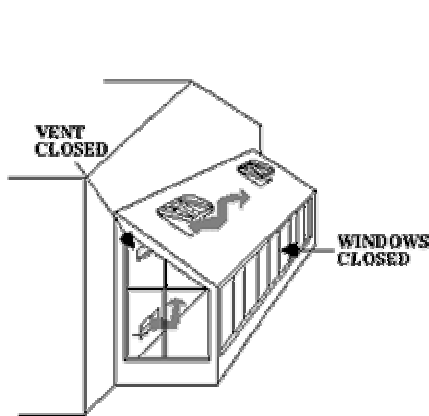


Figure 19. Summer venting sunroom

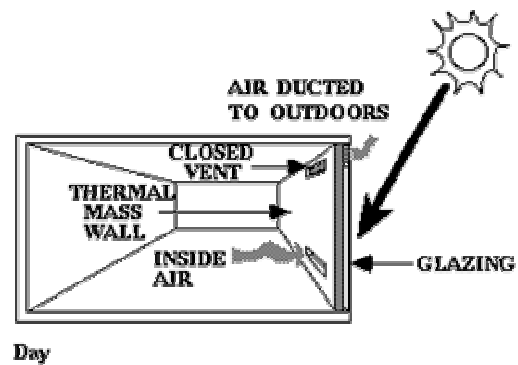


Figure 20. Summer venting thermal mass wall

Thermal mass indirect gain walls (Fig. 20) can be made to function similarly except that the mass wall should be insulated on the inside when performing this function.

Thermal chimneys (Fig. 21) can be constructed in a narrow configuration (like a chimney) with an easily heated black metal absorber on the inside behind a glazed front that can reach high temperatures and be insulated from the house. The chimney must terminate above the roof level. A rotating metal scoop at the top which opens opposite the wind will allow heated air to exhaust without being overcome by the prevailing wind.

Thermal chimney effects (Fig. 22) can be integrated into the house with open stairwells and atria. (This approach can be an aesthetic plus to the home as well.)

- Make the outlet openings slightly larger than the inlet openings. Place the inlets at low to medium heights to provide airflow at occupant levels in the room.
- Inlets close to a wall result in air "washing" along the wall. Be certain to have centrally located inlets for air movement in the center areas of the room.

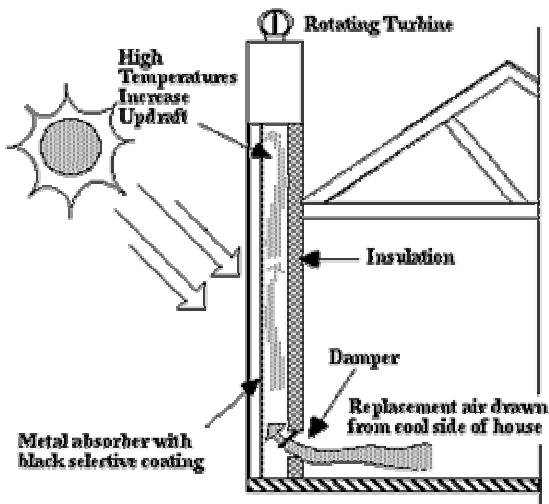


Figure 21. Thermal chimney

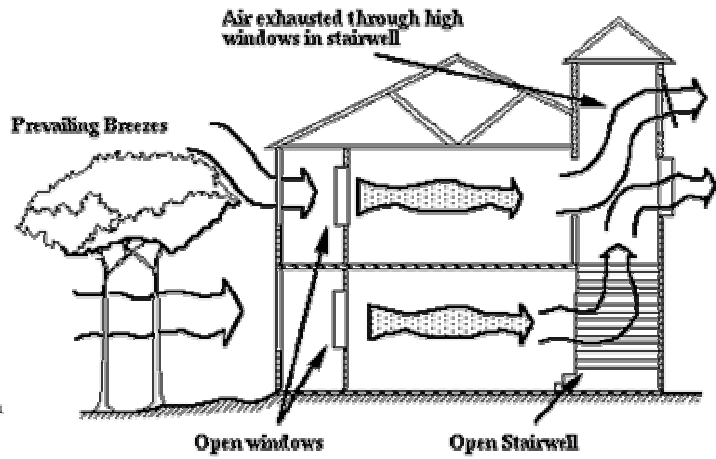


Figure 22. The rmalc himney effect built into home

Thermal mass is basically the ability of a material to store heat. It can be easily incorporated into a building as part of the walls and floor. Thermal mass affects the temperature within a building by:

- Stabilizing internal temperatures by providing heat source and heat sink surfaces for radiative, conductive and convective heat exchange processes.
- Providing a time-lag in the equalization of external and internal temperatures.
- Providing a reduction in extreme temperature swings between outside and inside.

Material selection to capitalize on thermal mass is an important design consideration. For instance, heavyweight internal construction (high thermal mass) such as brick, solid concrete, stone, or earth can store the Sun's heat during winter days, releasing the warmth to the rooms in the night after it conducts through. Lightweight materials such as plasterboard and wood paneling are relatively low mass materials and will act as insulators to the thermal mass, reducing its effectiveness. Lightweight construction responds to temperature changes more rapidly. It is therefore suitable for rooms that need to heat or cool very quickly.

For maximum energy efficiency, thermal mass should be maximized in the equator-facing sides of a building. Any heat gained through the day can be lost through ventilation at night. In using this technique, the thermal mass is often referred to as a 'heat bank' and acts as a heat distributor, delaying the flow of heat out of the building by as much as 10-12 hours.

Thermal mass design considerations include:

- Where mass is used for warmth, it should be exposed to incident solar radiation.
- Where mass is required for cooling, it is better placed in a shaded zone.

Buildings may be pre-cooled using night-purge ventilation (opening the building up to cool breezes throughout the night), although this requires significant amounts of exposed mass, and may be necessary only at certain times of the year.

Thermal mass is particularly beneficial where there is a big difference between day and night outdoor temperatures. High thermal mass with night ventilation relies on the daily heat storage of thermal mass combined with night ventilation that cools the mass. The building must be closed during the day and opened at night to flush the heat away.

Evaporative cooling lowers the indoor air temperature by evaporating water. In dry climates, this is commonly done directly in the space. But indirect methods, such as roof ponds, allow evaporative cooling to be used in more temperate climates too.

Ventilation and evaporative cooling are often supplemented with mechanical means, such as fans. Even so, they use substantially less energy to maintain comfort compared to refrigeration systems. It is also possible to use these strategies in completely passive systems that require no additional machinery or energy to operate.

Insulation specifications are another important design feature, [25]. Insulation reduces the rate at which heat flows through the building fabric, either outwards in winter or inwards in summer. In temperature controlled buildings, this will result in significant energy savings and increased thermal comfort. In passive buildings, it means that any low-grade energy available will be more effective at its job of heating or cooling.

Insulation has an additional benefit in that it also reduces noise transfer through the fabric; however its resistance to both fire and insects should also be major considerations. Proper installation is also essential to maximize performance, and there often local and international standards to cover the fire safety and health aspects of installation.

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