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NUMERICAL ANALYSIS OF FLUID FLOW AND HEAT TRANSFER IN A CYLINDRICAL CAVITY WITH AN OSCILLATING JET

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In the present study, a cylindrical cavity was considered. The cavity walls were kept at constant but different temperatures and an oscillating jet was applied through an orifice at the cold bottom end of the cavity to enhance the heat transfer on the hot top wall. This configuration was chosen because the heat transfer is low and enhancement could be desirable and easily identified. Jet velocity was oscillated sinusoidally in time and was uniform across the orifice cross section. A computer program was developed to investigate the characteristics of the fluid flow and heat transfer in the cavity. A parametric study of the effects of the injection velocity and frequency of oscillation on the heat removal from the hot top wall has been performed.

Geometry and coordinate system of the problem dealt herewith are given in Figure 1. The problem consists of a cylindrical enclosure of diameter D and height H. An oscillating jet is issued into the cavity through an orifice located at the centre of the bottom end of the cavity. The top end of the enclosure is kept at a constant but higher temperature than the constant temperature of the side wall and bottom wall. Air at temperature T_{in} is injected with velocity, $w_{in}=w_o \sin\omega t$ through the orifice.

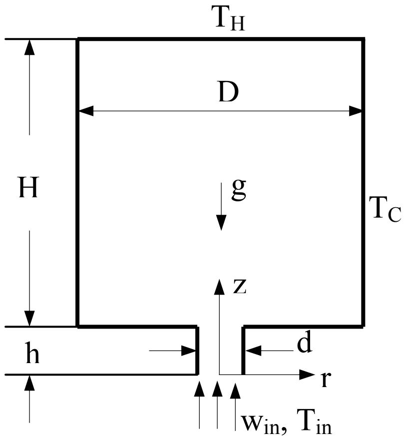

Figure 1. Geometry and coordinate system of the problem.

Since the injection velocity is oscillatory, the flow in the enclosure is unsteady. Also since the air is compressed and expanded in the enclosure during the process, the flow is compressible. For the injection velocities and wall temperature differences considered, the flow is assumed to be axi-symmetric and laminar. Depending on the values of Reynolds and Grashof numbers, the free convection and/or forced convection may play important roles on the flow and temperature fields. Therefore, the buoyancy should be included in the momentum equations. Under these conditions, the equations of the problem can be written as follow:

Continuity Equation

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial z}(\rho w)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho u)=0\,, \tag{1}$$

Momentum Equations

$$\frac{\partial}{\partial t}(\rho w)+\frac{\partial}{\partial z}(\rho wu)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho uw)=$$
$$-\frac{\partial p}{\partial z}+\frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial w}{\partial r}\right)+g_z\rho, \qquad (2)$$

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial z}(\rho wu)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho uu)=$$
$$-\frac{\partial p}{\partial r}+\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)-\mu\frac{u}{r^2}, \qquad (3)$$

Energy Equation

$$\frac{\partial}{\partial t}(\rho T)+\frac{\partial}{\partial z}(\rho wT)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho uT)=$$
$$-\frac{1}{c_v}Pdiv\vec{u}+\frac{1}{c_v}\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\frac{1}{c_v}\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) \qquad (4)$$

In the above equations, r and z represent the radial and axial directions. u is the radial component of the velocity, w is the axial component of the velocity, p is pressure and T is the temperature. ρ, k, μ and c_v are fluid density, thermal conductivity, dynamic viscosity and specific heat at constant volume, respectively. As k, μ and c_v were assigned constant values at average cavity temperature, the density ρ was assumed variable and calculated from ideal gas equation over the flow field.

Initially, in the enclosure, the fluid was assumed to be stationary and temperature was same as the surrounding temperature, T_C. The values of all the variables at inlet are prescribed. The axial component of the fluid velocity is specified as $w=w_o \mathrm{Sin}\omega t$. The radial component of the velocity at inlet is zero, i.e., u = 0. The temperature of the fluid at inlet was specified depending on the flow direction at the inlet. If the flow is into the cavity, the temperature is calculated as

$$T_{in}=T_a\left(\frac{P_{in}}{P_a}\right)^{\frac{k-1}{k}}$$

The inlet temperature calculated by this expression accounts for the increase in the temperature due to the isentropic compression of the air from atmospheric conditions to the conditions at the inlet. If the flow is outward from the cavity, the temperature is specified as the temperature at the adjacent control cells. All the walls have been assumed impermeable with no-slip condition. The top surface is kept at constant temperature T_H. The temperature on the cylindrical surface and at the bottom surface was assumed equal to the surrounding temperature, T_C. The flow was assumed to be symmetric about the axis of the cylinder, and hence, along the axis, the symmetry conditions hold.

Equations (1), (2), (3) and (4) have been solved together with relevant boundary conditions to determine the velocity and temperature distribution in the enclosure. A computer program based on the control volume formulation was developed.

The temperature distribution obtained from the numerical solution of the governing equations was used to calculate the Nusselt number along the hot surface. Heat flux from the hot surface to the fluid in the enclosure can be written as,

$$q''=h(T_w-T_C)=-k\left.\frac{\partial T}{\partial z}\right)_w$$

By rearranging this equation, we obtain the instantaneous local Nusselt number:

$$Nu_r = \frac{hD}{k} = -\frac{D}{(T_w - T_C)} \frac{\partial T}{\partial z}\Bigg)_w$$

Subscript r in the above expression indicates that the local Nusselt number on the hot top surface chances with coordinate direction r. Averaging the local Nusselt number over the area of hot surface and then averaging it over the period of oscillation (over a cycle), we obtain area averaged instantaneous Nusselt number and time averaged Nusselt number, respectively.

$$N\overline{u}_t = \frac{1}{A}\int_A Nu_r \, dA \qquad\qquad N\overline{u} = \frac{1}{\Delta t}\int_{\Delta t} \frac{1}{A}\int_A Nu_r \, dA\, dt$$

The computations were performed to analyse the effects of amplitude of injection velocity and oscillation frequency on the heat removal rate from the hot top wall. Keeping the Grashof number ($Gr = g\beta\Delta T D^3/\nu^2$) constant at 2.4×10^4 and the aspect ratio of the cavity (H/D) constant at 1, the simulations were repeated for Reynolds numbers ($Re = w_o D/\nu$) 255, 510, 767 and 1023, and for frequencies 1, 2, 4 and 8 Hz. The Nusselt numbers obtained for the cases with injection have been compared with Nusselt number of free convection (without any injection) case.

It was observed that the injection of the jet plays important roles on the flow field and heat transfer in the cavity. For the cases considered, the Nusselt number increases with increasing Reynolds number. The variation of the area averaged Nusselt number with Reynolds number during one cycle at oscillation frequency 1 is given in Fig. 2. As seen in this figure, at all Reynolds numbers, in the beginning of the injection period (inflow period) the Nusselt number is lower than the free convection Nusselt number. In the later part of the cycle the Nusselt number increases above the free convection Nusselt number. This variation of the Nusselt number is due to the effect of the jet on the flow field in the cavity. When the velocity field in the cavity is analysed, it is seen that in the beginning of the injection period, the effect of the jet does not reach to the top hot wall and it also retards the motion generated by the free convection in the upper part of the cavity. After about a quarter of the period, the effect of the jet penetrates up to the top hot wall. In the second half of the period (suction time), the motion due to the free convection is enhanced by the out flow. The computations revealed that the Nusselt number is not affected by the frequency of the oscillation for the cavity aspect ratio and Reynolds number considered. Again analysis of velocity field indicates that due to the shorter period of the injection at high frequencies, the effect of the jet does not diffuses to the top hot wall, and hence heat transfer is dominated by free convection. It should be pointed out here that with reducing aspect ratio this relation may change.

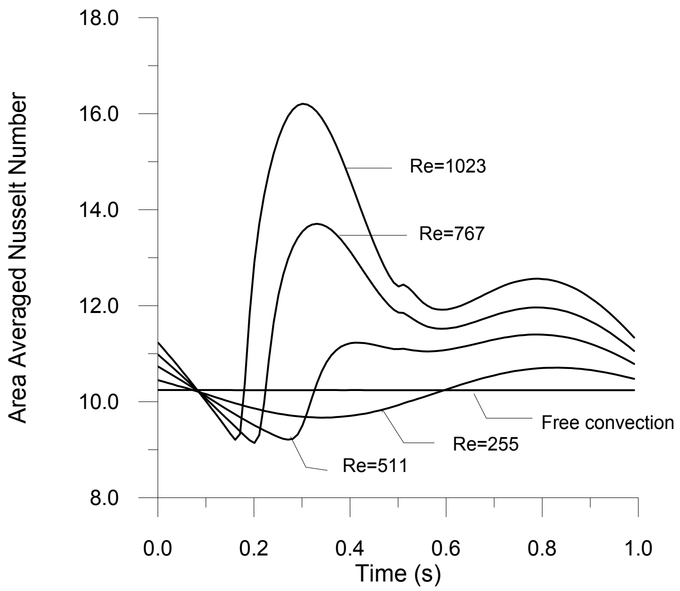

Fig 2. Variation of area averaged Nusselt number during one cycle for different Reynolds numbers ($Gr=2.4\times10^4$, H/D=1).