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FROM THE DIMENSIONLESS RESULTS TO THE PHYSICAL ONES IN TRANSIENT HEAT TRANSFER CONVECTION

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INTRODUCTION

Thermal systems, as heat exchangers or turbomachines, are often subjected to time-variations in the thermal boundary conditions (temperature or flux). Research of the evolutions in time of the temperature within the boundary-layer is generally conducted by one of the three privileged directions: direct numerical computation of the equations, extension of the differential method based on Blasius analysis[1,2,3], and Pohlhausen's integral method[4,5].

Direct numerical computation can be weighty if the momentum and energy equations are strongly coupled. The integral method present the inconvenient that it necessitates the knowledge of the velocity and temperature polynomial profiles in the boundaries layer. Moreover, both methods give the results specific to each physical problem (geometry, fluid, velocity, temperature, boundary conditions…).

Meanwhile, by an appropriate choice of the dimensionless quantities, the differential method can give the similarity solutions for some problems originally different. Thus, the non-dimensional results are, in most time, presented according to the dimensionless parameters. Whereas, a come back to the dimensional results, expressed in terms of physical parameters, can give some surprises and points out some phenomena not evidenced from the non-dimensional results.

This paper is an illustration of some discordance between the non-dimensional results and the physical ones, obtained from unsteady laminar forced convection.

APPLICATION

The problem considered here is the transient laminar forced convection from a flat plate (or a wedge) subjected to a step change in the surface temperature[2]. The pressure gradient is chosen to be different from zero, since flows with a non-constant velocity along the plate are often encountered in practical applications. The velocity field is assumed independent from temperature changes.

Using the dimensionless quantities:

$$\eta = \frac{y}{\sqrt{\nu x / U_\infty}}, \; U = U_\infty F'(\eta), \; m = \frac{x}{U_\infty}\frac{dU_\infty}{dx}, \; t^+ = \frac{U_\infty}{x}t, \text{ and } T^*(\eta, t^+) = \frac{T - T_\infty}{T_p - T_\infty} \tag{1}$$

the momentum equation can be reduced to:

$$F''' + \frac{m+1}{2}FF' + m\left(1 - F'^2\right) = 0 \tag{2}$$

and the transient energy equation becomes[2]:

$$\frac{1}{Pr}T^{*}{}'' + \frac{m+1}{2}FT^{*}{}' = \left[1 + (m-1)F't^+\right]\left(\frac{\partial T^*}{\partial t^+}\right) \tag{3}$$

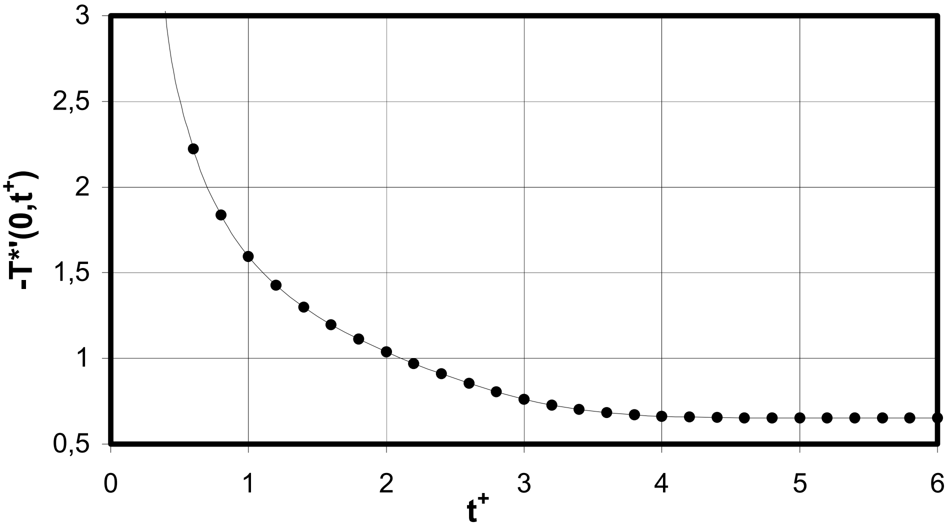

Fig. 1 - Gradient of dimensionless temperature for Blasius flow ($m = 0$)

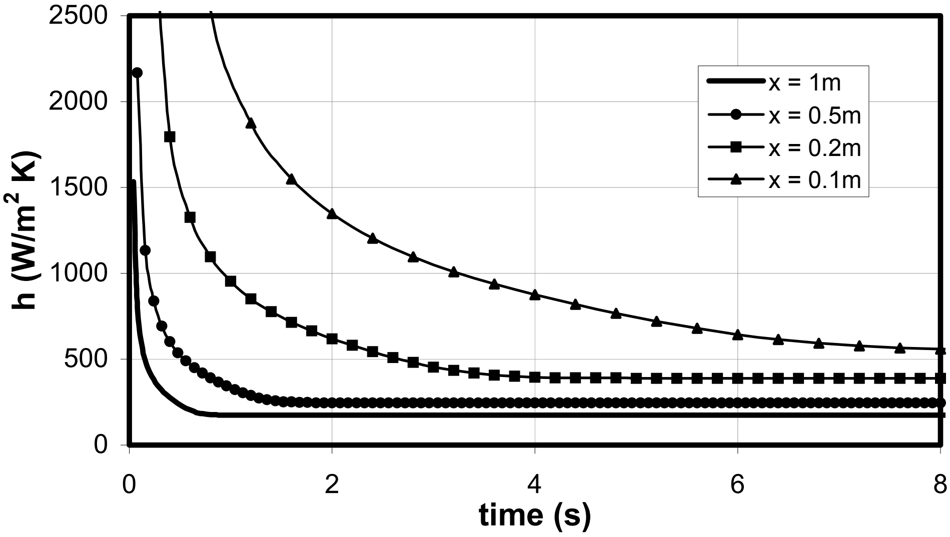

Fig. 2 – Instantaneous convective heat transfer coefficient at different abscissa ($m = 0$)

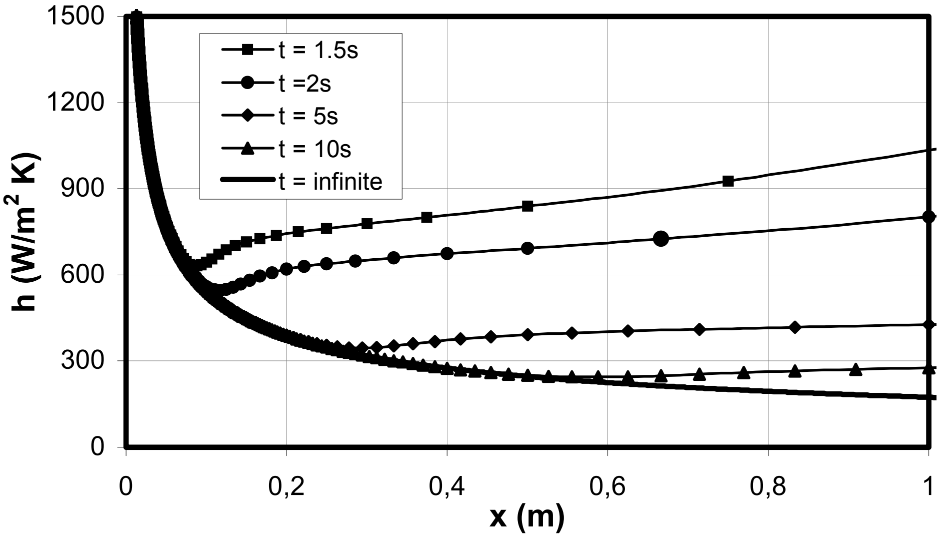

Fig. 3 – local convective heat transfer coefficient at different time ($m = 0$)

From the numerical resolution of the above equations (by Keller-Box method)[3] we obtain the dimensionless velocity and temperature, for each value of the pressure gradient parameter, *m*.

The local convective heat transfer coefficient for a uniform surface temperature is defined by:

$$h(x,t^+) = \frac{\varphi_p(x,t^+)}{(T_p - T_\infty)} = -T^{*\prime}(0,t^+)\,\lambda\sqrt{\frac{U_\infty}{\nu x}} \tag{4}$$

The evolution of the non-dimensional parameter $-T^{*\prime}(0,t^+)$, present in the expression of $h(x, t^+)$, is illustrated in *figure 1* for Prandtl number $Pr = 7$. From this evolution one can extract the physical evolutions of the convective heat coefficients in time (*figure 2*) and in space (*figure 3*).

The variation in time of the convective heat transfer coefficient at a given location x (*figure 2*) is similar to the variation of $-T^{*\prime}(0,t^+)$ with t^+.

The physical evolutions of the convective heat transfer with the axial coordinate, x, at different instants t (*figure 3*) show behaviour in unsteady state different from which occurs in steady state (t = infinite). Indeed, in steady state the convective heat coefficient presents a monotonous decrease with x from an infinite value to an asymptotic one. However, the transient coefficient $h(x, t)$ doesn't have a monotonous variation with x. It diminishes near the leading edge of the plate and slowly arises in the region at far distance from the entrance. Mathematically, this is due to the presence of the space variable, x, in both expressions of $h(x, t^+)$ and t^+ *(see equation (1))*. Physically, the difference between these two behaviours is due to the fact that the fluid is not heated (if $T_p > T_\infty$) in steady state on the same distance as in transient state. In steady state, the heat exchange at a location (x) occurs between a plate at T_p and a fluid heated from the leading edge of the plate ($x = 0$) until the location (x). So, it has a temperature near the plate close to T_p. On the same location (x), in the first instants after the impulsively change of the plate temperature from T_p to T_∞, the fluid temperature near the plate is not heated from the entrance, so its temperature is close to T_∞. Thus, the heat flux extracted from the plate, therefore, the convective heat transfer coefficient, is greater in the second case than in the first case (since $\varphi_p = h(T_p - T_\infty)$ and $T_p - T_\infty$ is the same in steady and unsteady states).

Other illustrations of the importance of the dimensional representation of results are also given in this paper. We show, in particular, the interpretation errors which might be induced by the representation of the dimensionless convective heat coefficient and the transient duration as a function of the Prandtl number.

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