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AN EFFICIENT NODAL SCHEME FOR TIME-DEPENDENT CONVECTIVE HEAT AND MASS TRANSFER IN SINGLE-PHASE FLOWS

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Transient, coupled heat and mass transfer problems still pose a challenge to available computational resources. In addition, evaluation of numerical schemes for such problems is hampered by lack of analytical solutions in more than one dimension. Coarse mesh nodal methods have been highly successful for neutronics calculations. However, this level of success has not been duplicated in other branches of science and engineering. We will report the results of our recent efforts toward the development of an efficient nodal scheme to solve set of coupled, two-dimensional, time-dependent Navier-Stokes, energy and concentration equations. Coupling between the Navier-Stokes and energy and concentration equations is via the Boussinesq approximations. In addition, we present an analytical solution to a modified two-dimensional, lid driven cavity problem with heat and mass transfer. Comparison of results obtained using the nodal scheme with the exact analytical results will also be presented.

Despite several excellent attempts [1-6], applications of nodal schemes to areas outside neutronics—especially outside the nuclear community—have been slow due to several reasons. To address some of these limitations our recent efforts have focused on the development of: **1)** a modified nodal method for the time-dependent Navier-Stokes (N-S) equations and its parallel implementation [7,8]; **2)** hybrid methods for domains with curved boundaries [9,10]; and **3)** adaptive mesh refinement (AMR) capability for nodal schemes [10,11].

The modified nodal method for the time-dependent, incompressible Navier-Stokes (N-S) equations incorporates two major modifications over nodal schemes developed earlier. First, rather than using the conventional continuity equation [1,4] or the vorticity-stream function formulation [2] which is difficult to extend to three dimensions, we replace the conventional continuity equation by a Poisson-type continuity equation written in terms of pressure, and retain the momentum equations in primitive variables. The second modification [12] is introduced in the development of the numerical scheme. Here, rather than using only the diffusion term to obtain the homogeneous part of the solution of the momentum equations [1,4], a "linearized" convection term—based on previous time step velocity—is also retained on the left hand side of the transverse-integrated equations leading to a local homogeneous solution for the transverse-integrated velocities in each spatial direction which is a combination of a constant, a linear and an exponential term.

The set of N-S-energy-specie (N-S-E-S) equations is

$$\nabla^2 p(x,y,t) + \rho\left(\frac{\partial u}{\partial x}\right)^2 + 2\rho\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \rho\left(\frac{\partial v}{\partial y}\right)^2 = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u(x,y,t) + g\sin(\theta)\, h(T(x,y,t), C_i(x,y,t))$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v(x,y,t) + g\cos(\theta)\, h(T(x,y,t), C_i(x,y,t))$$

$$\rho c_p\left[\frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T\right] = k\nabla^2 T(x,y,t) + q_T(x,y,t)$$

$$\left[\frac{\partial C_i}{\partial t} + \vec{v}\cdot\nabla C_i\right] = D_i\nabla^2 C_i(x,y,t) + q_i(x,y,t)$$

where C_i is the concentration of specie i ($i = 1, \ldots, K$), K is the total number of species, and q_T and q_i are the energy and specie source terms. The momentum equations depend upon temperature and/or specie concentrations via the Boussinesq approximation,

$$h = (1 - \beta_T(T(x,y,t) - T_\infty) - \sum_i \beta_{Ci}(C_i(x,y,t) - C_{i\infty}))$$

Application of the nodal approach leads to a difference scheme with node interior variations for the transverse integrated quantities that are quadratic for pressure, and of the form *(constant + linear + exponential)* for the velocities, temperature and specie concentration.

We also present an exact solution for the coupled set of N-S-E-S equation—based on one proposed earlier [13] for the N-S equations. An exact solution of the coupled, steady-state N-S-E-S equations for a modified lid driven cavity problem *($0 < x, y < 1$)* with $C_{i\infty} = T_\infty = \theta = 0$, $\beta_T = \beta_{Ci}$, is given by

$$u(x,y) = 8(x^4 - 2x^3 + x^2)(4y^3 - 2y)$$

$$v(x,y) = -8(4x^3 - 6x^2 + 2x)(y^4 - y^2)$$

$$p(x,y) = 8\nu[F(x)g'''(y) + f'(x)g'(y)] + 64F_2(x)\{g(y)g''(y) - [g'(y)]^2\}$$

$$T(x,y) = \gamma_0[1 + b(x,y)/g]/\beta_T \qquad and \qquad C_i(x,y) = \gamma_i[1 + b(x,y)/g]/\beta_{Ci}$$

where γ_j are weight factors such that their sum is equal to one,

$$b(x,y) = -8\nu[24F(x) + 2f'(x)g''(y) + f'''(x)g(y)] - 64[F_2(x)G_1(y) - g(y)g'(y)F_1(x)]$$

$$f(x) = (x^4 - 2x^3 + x^2); \quad g(y) = (y^4 - y^2); \quad F(x) = \int f(x)dx;$$

$$F_1(x) = f(x)f''(x) - [f'(x)]^2; \quad F_2(x) = 0.5[f(x)]^2; \quad G_1(y) = g(y)g'''(y) - g'(y)g''(y)$$

$$f(x) = x^4 - 2x^3 + x^2;\ g(y) = y^4 - y^2;\ F(x) = \int f(x)dx;\ F_1(x) = f(x)f''(x) - [f'(x)]^2;\ F_2(x) = 0.5[f^2(x);$$
$$G_1(y) = g(y)g'''(y) - g'(y)g''(y)$$

and energy source term is given by

$$q_T(x,y) = \rho c_p \vec{v}\cdot\nabla T - k\nabla^2 T(x,y)\cdot$$

The specie source term is given by the corresponding specie equation. The flow in the cavity is due to shear as well as due to body forces caused by energy and specie sources/sinks. Note that the lid velocity is not uniform and is given by

$$u(x, y=1) = 16(x^4 - 2x^3 + x^2) \cdot$$

The nodal scheme described above for the time-dependent N-S-E-S equations is used to solve the lid driven cavity with thermal and specie sources/sinks. The steady-state problem was solved by marching in time starting from a spatially uniform initial condition for all variables. Gauss-Seidel iterations are used at each time step. Numerical results for this and other problems will be presented.

REFERENCES

1. Y.Y. Azmy and Dorning, "A Nodal Integral Approach to the Numerical Solution of Partial Differential Equations," *in Advances in Reactor Computations*, volume II, pages 893-909, American Nuclear Society, LaGrange Park, IL, 1983.
2. P. D. Esser, R. J. Witt, “An Upwind Nodal Integral Method for Incompressible Fluid flow,” *Nuclear Science and Engineering*, **114**, 20-35(1993)
3. O.A. Elnawawy, A.J. Valocchi and A.M. Ougouag, "The Cell Analytical Numerical Method for Solution of the Advection-Dispersion Equation: Two-Dimensional Problems," Water Resources Research, 26, 11, 2705-2716, 1990.
4. G.L. Wilson, R.A. Rydin, Y.Y. Azmy, "Time-Dependent Nodal Integral Method for the Investigation of Bifurcation and Nonlinear Phenomena in Fluid Flow and Natural Convection," *Nucl. Sci. Eng.,* **100**, 414-425, 1988.
5. J.P.Hennart, “*A General Family of Nodal Schemes*,” *SIAM J. Sci. Stat. Comp*., **7,** 1, 264-287 (1986).
6. E. P. E. Michael, J. J. Dorning, E. M. Gelbard, Rizwan-Uddin, “A Nodal Integral Method for the Convection-Diffusion Heat Equation”, *Trans. Am. Nucl. Soc.*, **69**, 239-241, 1993.
7. F. Wang, Rizwan-Uddin, “A Nodal Scheme for the Time-Dependent Incompressible Navier-Stokes Equations”, *Trans. Am. Nucl. Soc*., **83**, 422-424, 2000.
8. F. Wang, “An Advanced Numerical Scheme for the Time-Dependent, Incompressible, Navier-Stokes-Energy-Concentration Equations and its Parallel Implementation,” Ph.D. Thesis, University of Illinois, in preparation.
9. Allen J. Toreja and Rizwan-uddin, “Hybrid Numerical Methods for the Convection-Diffusion Equation in Arbitrary Geometries”, *Proceedings of the International Conference on Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications*, J. M. Aragones, C. Ahnert, O. Cabellos (Eds), 1705-1714, Senda Editorial, Madrid, 1999.
10. A. Toreja, A Nodal Approach to Arbitrary Geometries, and Adaptive Mesh Refinement for the Nodal Method, Ph.D. Thesis, University of Illinois, 2002.
11. Allen J. Toreja and Rizwan-uddin, “Adaptive Mesh Refinement for the Nodal Integral Method and Application to the Convection Diffusion Equation,” *Trans. Am. Nuc. Soc*., **84**, 182-185 (2001).
12. B.L. Wescott and Rizwan-uddin, “An Efficient Formulation of the Modified Nodal Integral Method and Application to 2-D Burgers’ Equation,” to appear in *Nucl. Sci. & Engg*., November, 2001.