TEMPORAL EVOLUTION OF THERMAL CONVECTION IN AN INITIALLY, STABLY STRATIFIED FLUID

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The temporal growth of buoyancy-driven convection in an initially quiescent, stably stratified fluid layer confined between the two horizontal plates is investigated theoretically. In this time-dependent, developing temperature field the initial conditions of disturbances are forced, based on the available results based on propagation theory. The Oberbeek-Boussinesq equations are solved by using the finite element method (FEM) and a new parameter to mark the onset of fastest growing instabilities is proposed. Also, the characteristic time to represent manifest convection is examined in comparison with existing experimental data.

ONSET OF CONVECTIVE INSTABILITY

Convective instabilities in a horizontal layer of Newtonian fluid have been investigated extensively sinc e 1900. But the instability problem in a developing, nonlinear temperature field is still clouded because of its inherent complexity. Morton[1], Foster[2], Jhavery and Homsy[3], Tan and Thorpe[4], and Yang and Cho i[5] have conducted the related instability analysis by using the frozen-time model, amplification theory, stochastic model, maximum-Rayleigh-number criterion and propagation theory, respectively.

The system considered here is an initially stably stratified, horizontal fluid layer of thickness L, as sho wn in Fig. 1. For time $t<0$, the conduction field has a linear temperature profile with $T=T_i$ at the vert ical distance $Z=0$ and $T=T_u$ at $Z=L$. The fluid layer is heated with a higher temperature T_b for tim e $t \geq 0$. The important parameters to describe the present system are the Prandtl number $Pr\ (=\nu/\alpha)$, the Rayleigh number $Ra\ (=g\beta(T_b-T_i)L^3/\alpha\nu)$ and the temperature ratio $\gamma\ (=(T_u-T_i)/(T_b-T_i))$. Her e ν denotes the kinematic viscosity, α the thermal diffusivity, g the gravitational acceleration, and β the thermal expansivity. When the fluid layer is heated slowly, the temperature profile becomes linear. I t is well-known that with thermal convection the Rayleigh number have to exceed the value of $Ra(1-\gamma)=1708$ for $\gamma<1$ under the constant temperature gradient. However, most of actual processes involve nonlinea r, developing temperature profiles. Therefore, it is important to find the characteristic time to mark the o nset of thermal convection for a given Ra and Pr. Under thermal convection the dimensionless gover ning equations of flow and temperature fields are expressed as follows:

$$\left\{\frac{\partial}{\partial\tau}+\boldsymbol{u}\cdot\nabla\right\}\boldsymbol{u}=-\nabla p+Pr\nabla^2\boldsymbol{u}+Pr\,Ra\,\theta\,\boldsymbol{k} \tag{1}$$

$$\left\{\frac{\partial}{\partial\tau}+\boldsymbol{u}\cdot\nabla\right\}\theta=\nabla^2\theta \tag{2}$$

where $\theta\ (=(T-T_i)/(T_b-T_i))$, $p\ (=PL^2/\rho\alpha^2)$, $\tau\ (=\alpha t/L^2)$ and $\boldsymbol{u}\ (=\boldsymbol{U}/(\alpha/L))$ denote the dimensi onless forms of the temperature T, the pressure P, the time t, and the velocity vector $\boldsymbol{U}$, respectiv ely. Here $\boldsymbol{k}$ represents the vertical unit vector. The velocity satisfies the continuity equation $(\nabla \cdot \boldsymbol{u} = 0)$ and the Cartesian coordinates are nondimensionalized by L.

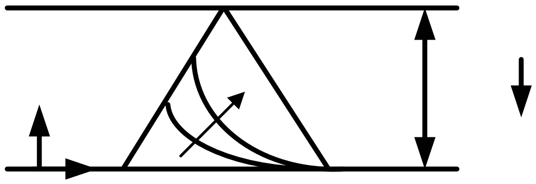

Fig. 1. Temperature profiles in conduction state.

Self-similar transformation

Propagation theory is based on the assumption to the effect that the temperature disturbances to mark the onset of convective instability are propagated mainly within the thermal penetration depth at the onset time. For the present system Kim et al.[6] employed the scale analysis and transformed the governing linearized equations into the self-similar forms. Their characteristic time to mark the onset of thermal instability, i.e., t_c^*, compares favorably with Ueda et al.'s[7] experimental data of $\gamma Ra^{-1/3} < 0.025$, considering the growth period of disturbances until they are detected.

Numerical simulation

The initial values of velocity and temperature disturbances with the proper magnitude forced are assumed, based on the above predictions[6]. They have the shape of cells aligned with the horizontal. In the present study, with the concept of mean field approximation, the dimensionless temperature θ is expressed as $\theta = \langle\theta\rangle + \theta'$, where $\langle\theta\rangle$ is the horizontal average temperature and θ' is the temperature fluctuation. The time-dependent growth rates of mean temperature and thermal disturbances are given, as shown in Table 1. Here the rms (root mean squared) value of property ϕ is defined as $\phi_{rms} = [(\int_A \phi^2 dA)/A]^{1/2}$, where A is the horizontal area.

Table 1. Growth rates of temperature.

Growth rate of mean temperature	Growth rate of temperature disturbances
$r_0 = \dfrac{1}{\langle\theta\rangle_{rms}} \dfrac{d\langle\theta\rangle_{rms}}{d\tau}$	$r_1 = \dfrac{1}{\theta'_{rms}} \dfrac{d\theta'_{rms}}{d\tau}$

The numerical simulation was conducted by solving Eqs. (1) and (2) with the FEM. The values of vertical velocity disturbance w_{rms} and the temperature disturbance θ'_{rms} are almost the same values as those of the initial disturbances for a short time. But at a certain time they experience a sudden increase and show the maximum before they decrease. Their growth rates are illustrated in Fig. 3, wherein r_0 is nearly the same as that of conduction temperature. For small τ, r_1 increases from the negative value to weaken the initial disturbances. At small time when the conduction heat transfer is dominant, r_0 is larger than r_1. With the growth of disturbances r_1 exceeds r_0. The critical time τ_c to mark the onset of convective instability is assumed to be the characteristic time at $r_1 = r_0$. This time is a fixed value which is not changed by magnitude of the initial conditions. The present critical time τ_c is near Kim et al.'s[6] critical time τ_c^*.

MANIFEST CONVECTION

For $r_0 > r_1$ fluctuations may remain unobservably small at experimental environments. Ueda et al.[7] observed convective motion at $\tau = \tau_m$ in experiments of $\gamma = 0.73 \sim 1.67$, $Pr = 8800$ and $Ra = 9000 \sim 17000$. They used aluminum powders for flow visualization. Their data points are shown in Fig. 2, which represent the onset of manifest convection. This means that manifest convection is detected at

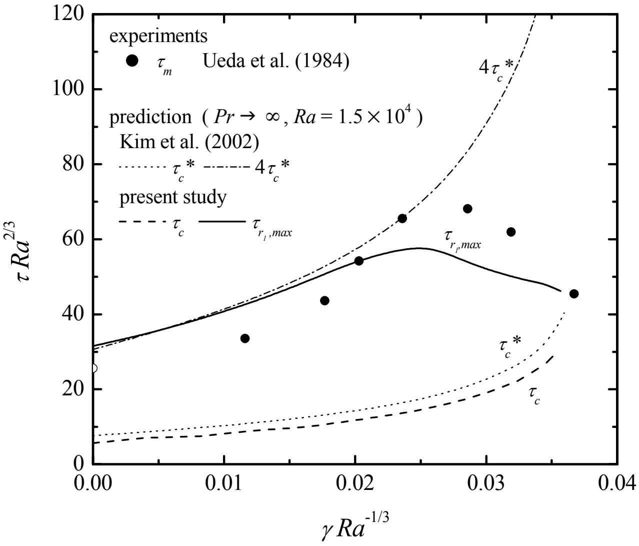

Fig. 2. Comparison with experiments.

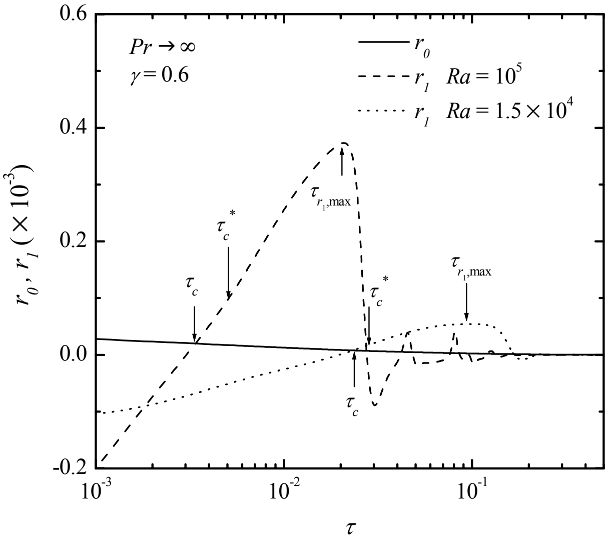

Fig. 3. Temporal behavior of growth rates.

the detection time $\tau_m \cong 4\tau_c^*$ for small γ. In their whole experimental range the data points are located near the charactersitc time $\tau_{r_1,\max}$, when r_1 reaches the maximum value. Therefore it may be stated that the growth period is required from the onset time of a fastest growing instability, i.e., τ_c, to $\tau = \tau_{r_1,\max}$. Up to the latter time linear theory is valid. With time thermal convection becomes fully developed, as shown in Fig. 3 and with increasing Ra the growth period decreases.

CONCLUSION

The critical time to mark the onset of thermal instability has been investigated by using the FEM. It is reported here that a fastest growing mode of regular cells would set in at $\tau = \tau_c$ with $r_0 = r_1$ and they grow until manifest convection is detected at $\tau = \tau_{r_1,\max}$. Up to the latter characteristic time linear theory seems to be valid. For $\tau < \tau_c$ the fluctuations seem to be negligible noise. The present analysis overcomes the limitations of the previous stability analyses, to a certain degree.

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