ONSET OF CONVECTIVE MOTION IN A HORIZONTAL FLUID LAYER HEATED FROM BELOW AND COOLED FROM ABOVE WITH CONSTANT HEAT FLUX

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A theoretical analysis of thermal instability driven by buoyancy forces in transient temperature fields is conducted in an initially quiescent, horizontal fluid layer heated from below and cooled from above with uniform heat flux. Under the principle of the exchange of stabilities, the stability analysis is performed on the basis of the propagation theory which adopts the thermal boundary-layer thickness as a characteristic length scaling factor and self-similar transformation. The prediction compares quite well with existing experimental result.

THEORETICAL ANALYSIS

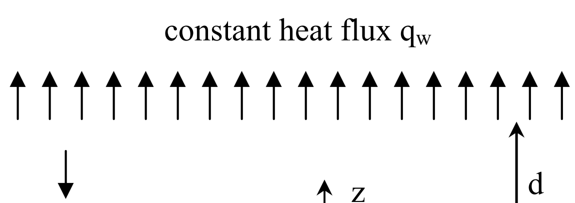
When an initially quiescent fluid layer confined between two horizontal plates is heated rapidly from below and/or cooled rapidly from above, the basic temperature profiles of heat conduction develop with time and buoyancy-driven convection can set in at a certain time. In this transient system the critical time t_c to mark the onset of convective motion becomes an important question. The related instability analysis has been conducted by using the frozen-time model[1], propagation theory[2], maximum-Rayleigh-number criterion[3] and amplification theory[4]. The first two models are based on linear theory and yield the critical time as the parameter. The last model requires the initial conditions at the heating time $t = 0$ and the criteria to define manifest convection.

Mathematical formulation

The problem considered here is a horizontal fluid layer confined between two rigid boundaries separated by a distance "d", as shown in Fig. 1. The fluid layer is initially motionless at a constant temperature T_0. For time $t \geq 0$, the fluid layer is heated from below and cooled form above with constant heat flux q_w. The schematic diagram of the basic system of pure conduction is shown in Fig. 1. The dimensionless basic temperature θ_0 has the scale of $(q_w d)/k$, where k denotes the thermal conductivity. Under linear stability theory, the disturbances caused by the onset of thermal convection can be formulated, in dimensionless form, in terms of the temperature component θ_1 and the vertical velocity component w_1 as

$$\left\{\frac{1}{\Pr}\frac{\partial}{\partial\tau} - \nabla^2\right\}\nabla^2 w_1 = -\nabla_1^2\theta_1\,, \tag{1}$$

$$\frac{\partial\theta_1}{\partial\tau} + \mathrm{Ra}_q\, w_1 \frac{\partial\theta_0}{\partial z} = \nabla^2\theta_1\,, \tag{2}$$



rigid

g

rigid

constant heat flux q_w

Fig. 1. Schematic diagram of system considered here.

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ and $\nabla_1^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$. The dimensionless velocity component has the scale of α/d, the vertical distance d, and the temperature disturbance the scale of $\alpha\nu/(g\beta d^3)$, where α denotes the thermal diffusivity, ν the kinematic viscosity, g the gravity acceleration and β the thermal expansivity. Here $\mathrm{Ra}_q\left(= g\beta q_w d^4/(k\alpha\nu)\right)$ is the Rayleigh number, $\tau\left(= t\alpha/d^2\right)$ the dimensionless time, and $\mathrm{Pr}(=\nu/\alpha)$ the Prandtl number. The proper boundary conditions are given by

$$w_1 = \partial w_1/\partial z = \partial\theta_1/\partial z = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1. \tag{3}$$

Our goal is to find the critical time τ_c to mark the onset of convective instability for a given Ra_q by using Eqs. (1)-(3). With the frozen-time model, the term involving $\partial(\cdot)/\partial\tau$ is neglected and therefore the system becomes time-independent, the results are independent of Pr. With the amplification theory the proper initial conditions at $\tau = 0$ and the amplification factor to represent manifest convection are required. However, propagation theory is a rather simple, deterministic approach even though it involves the transient effect.
The present study will employ propagation theory for the stability analysis. Based on scaling analysis, the following amplitude relations can be obtained:

$$|w_1/\theta_1| \sim \delta_T^2 \sim \tau, \quad \mathrm{Ra}^*\left(w_1/\delta_T^2\right)D\theta_0 \sim \theta_1. \tag{4a,b}$$

where $\mathrm{Ra}^* = \mathrm{Ra}_{\Delta_T}$ is the Rayleigh number based on the thermal boundary-layer thickness Δ_T and the wall heat flux q_w, and $\delta_T\left(\propto \tau^{1/2}\right)$ is the nondimensional thermal boundary-layer thickness. In propagation theory, $\mathrm{Ra}^*\left(= \mathrm{Ra}_q\tau^2\right)$ is assumed to be a constant. From the resulting perturbation equations the self-similar stability equations[2] are obtained and the minimum value of Ra^*, i.e., Ra_c^* is obtained numerically.

Stability analysis results

The stability criteria for various Pr values are summarized in Fig. 2. It seems evident that the critical Rayleigh number Ra_c^* increases with a decrease in Pr and the Pr-effect becomes pronounced for $\mathrm{Pr} < 1$. For the high-Pr case, the long wave mode is preferred. This means that the inertia forces make the system more stable and the disturbance be confined within the narrow regions near the boundary.

Ward and Le Blanc[5] measured the voltage variation with time in an electrochemical redox system where the Schmidt number Sc, which is equivalent to Pr in a heat transfer system, is larger than 1,000. For this electrochemical system the present τ_c -

values are about one sixth of the experimental result τ_o, as shown in Fig. 3. Here τ_o represents the characteristic time to mark manifest convection. Foster[6] commented that with correct dimensional relations $\tau_o \cong 4\tau_c$. This means that a fastest growing mode of instabilities, which set in at $t = t_c$, will grow with time until manifest convection is detected at $t_o \cong 4t_c$. A growth period will be required, as illustrated in Fig. 3. The validity of $t_o \cong 4t_c$ requires a further study but this relation is shown even in other transient diffusive systems[2].

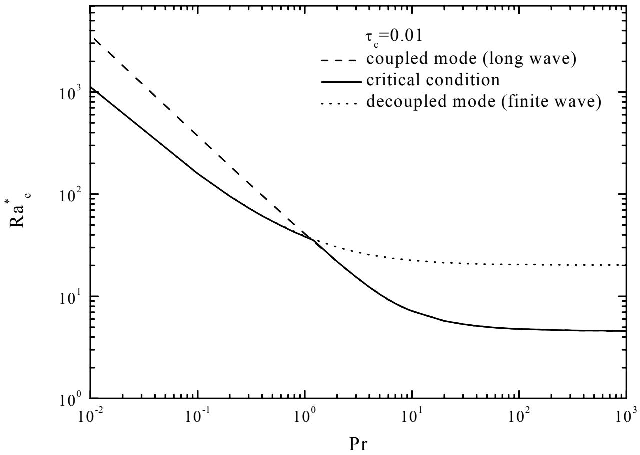


Fig. 2. Effect of Pr on critical condition.

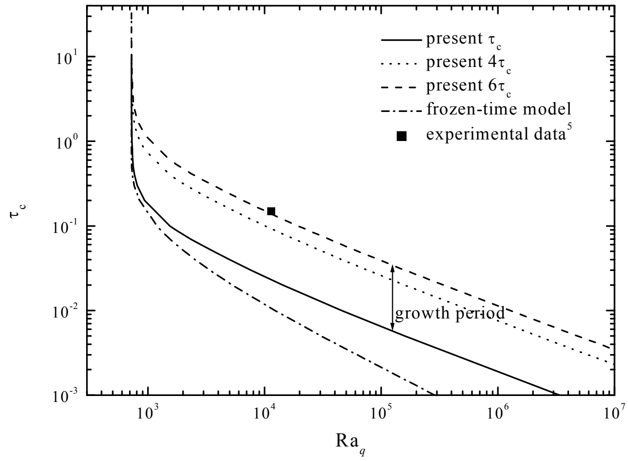


Fig. 3. Comparison of theoretical results with experimental one.

CONCLUSION

The onset of buoyancy-driven motion in a horizontal fluid layer heated from below and cooled from above with uniform heat flux has been analyzed analytically by using linear stability theory. The propagation theory and frozen-time model predict that the onset time of buoyancy driven motion is a function of the Rayleigh number and the Prandtl number. It seems that for high Pr case, the long wave mode is preferred and the inertia forces make the system more stable and the disturbance be confined within the narrow region. The present result shows that the propagation theory predicts the onset time quite well and the inertia term makes the system more stable.

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