NUMERICAL ANALYSIS OF LAMINAR MIXED CONVECTION FLOW INSTABILITIES IN A VERTICAL PIPE

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Recent developments in engineering have led to an increasing interest in transient mixed convection flows. These flows arise in many technological applications such as nuclear plants, heat exchangers and other apparatuses, etc. The unsteady characteristics of the flow can lead to the flow reversal therefore unstable. Martin *et al.* [1] have observed this phenomenon in the case of mixed convection through vertical tube at high Grashoff numbers. Gau *et al*. [2] have experimentally studied the reversed flow structure and heat transfer in a finite asymmetrically heated vertical channel. The oscillation of flow induced by the reversed flow is visualized. However, almost all the work rests on the combined effects between free and forced convection through vertical tubes, when a sudden change in the wall temperature or the wall heat flux. Very few studies are available on the thermohydraulic structures in a vertical pipe flow, when the inlet boundary conditions are changing with time. In this situation, the present paper investigates the transient mixed convection in an upward vertical pipe flow, when a temperature step is imposed at the entrance. The positive and negative temperature step effects and the wall-to-fluid heat capacity ratio on the nature of the transient flow behaviour proved to be of special interest.

The equations considered are the two-dimensional incompressible continuity, momentum, and energy equations in axis-symmetric coordinates. The governing equations are formulated in terms of the stream function and the vorticity:

$$U^+ = -\frac{1}{r^+}\frac{\partial \psi^+}{\partial z^+} \quad ; \quad V^+ = \frac{1}{r^+}\frac{\partial \psi^+}{\partial r^+}$$

$$\Omega^+ = \frac{\partial U^+}{\partial z^+} - \frac{\partial V^+}{\partial r^+}$$

$$\frac{\partial \Omega^+}{\partial t^+} + \frac{\partial\left(\Omega^+ V^+\right)}{\partial z^+} + \frac{\partial\left(\Omega^+ U^+\right)}{\partial r^+} = \frac{1}{R_e}\left[\frac{1}{r^+}\frac{\partial}{\partial r^+}\left(r^+\frac{\partial \Omega^+}{\partial r^+}\right) + \frac{\partial^2 \Omega^+}{\partial z^{+2}}\right] + Ri\frac{\partial T^+}{\partial r^+}$$

$$-\Omega^+ = \frac{\partial}{\partial r^+}\left(\frac{1}{r^+}\frac{\partial \psi^+}{\partial r^+}\right) + \frac{1}{r^+}\frac{\partial^2 \psi^+}{\partial z^{+2}}$$

$$\frac{\partial T^+}{\partial t^+} + \frac{1}{r^+}\frac{\partial\left(r^+ U^+ T^+\right)}{\partial r^+} + \frac{\partial\left(V^+ T^+\right)}{\partial z^+} = \frac{1}{Pe}\left[\frac{1}{r^+}\frac{\partial}{\partial r^+}\left(r^+\frac{\partial T^+}{\partial r^+}\right) + \frac{\partial^2 T^+}{\partial z^{+2}}\right]$$

These equations are subject to specific boundary conditions :

- (at the entrance) $z^+ = 0$: $\psi^+ = r^+\left(1 - \frac{r^{+2}}{2}\right) - \frac{1}{2}$; $\Omega^+ = 4r^+$; $T^+ = 1 + \Delta T^+$

where $\Delta T^+ = \frac{\Delta T_e}{T_e - T_a}$ ($\Delta T^+ > 0$ or < 0)

- (centre line) $r^+ = 0$: $\frac{\partial T^+}{\partial r^+} = \frac{\partial \psi^+}{\partial r^+} = 0$; $\Omega^+ = 0$

- (at the wall) $r^+ = 1$: $\psi^+ = 0$; $\left(\frac{\partial T^+}{\partial r^+}\right)_w = 1 - a^* Pr\frac{\partial T^+}{\partial t^+}$

where $a^* = \frac{(\rho C_p S)_w}{(2\rho C_p S)_f}$: wall-to-fluid heat capacity ratio.

The equations are finite-differenced in an explicit scheme similar to that used by Torrance *et al*. [3].

In this work, the following conditions are selected in the computations: water flow, copper pipe of diameter 20/22 mm, the bulk velocity flow is 0.01 m/s. In the initial state, the temperature of the fluid is uniform and the flow is fully developed. A positive or negative temperature step is suddenly applied at the entrance of the pipe.

Figure 1 shows the time development of the recirculating flow near the wall in the case of positive temperature step. After a sudden perturbation at the entrance of the tube, as can be seen from the figure, a vortex begins to form in the vicinity of the wall at t = 2 s. As time progress, the multi-cellular propagates along the wall. This leads to the wall boundary layer instability. This phenomenon is due to the transient mixed convection flows. The influence of the transient boundary conditions at the entrance of the pipe and the wall-to-fluid heat capacity ratio on the laminar flow behaviour will be analysed in detail.

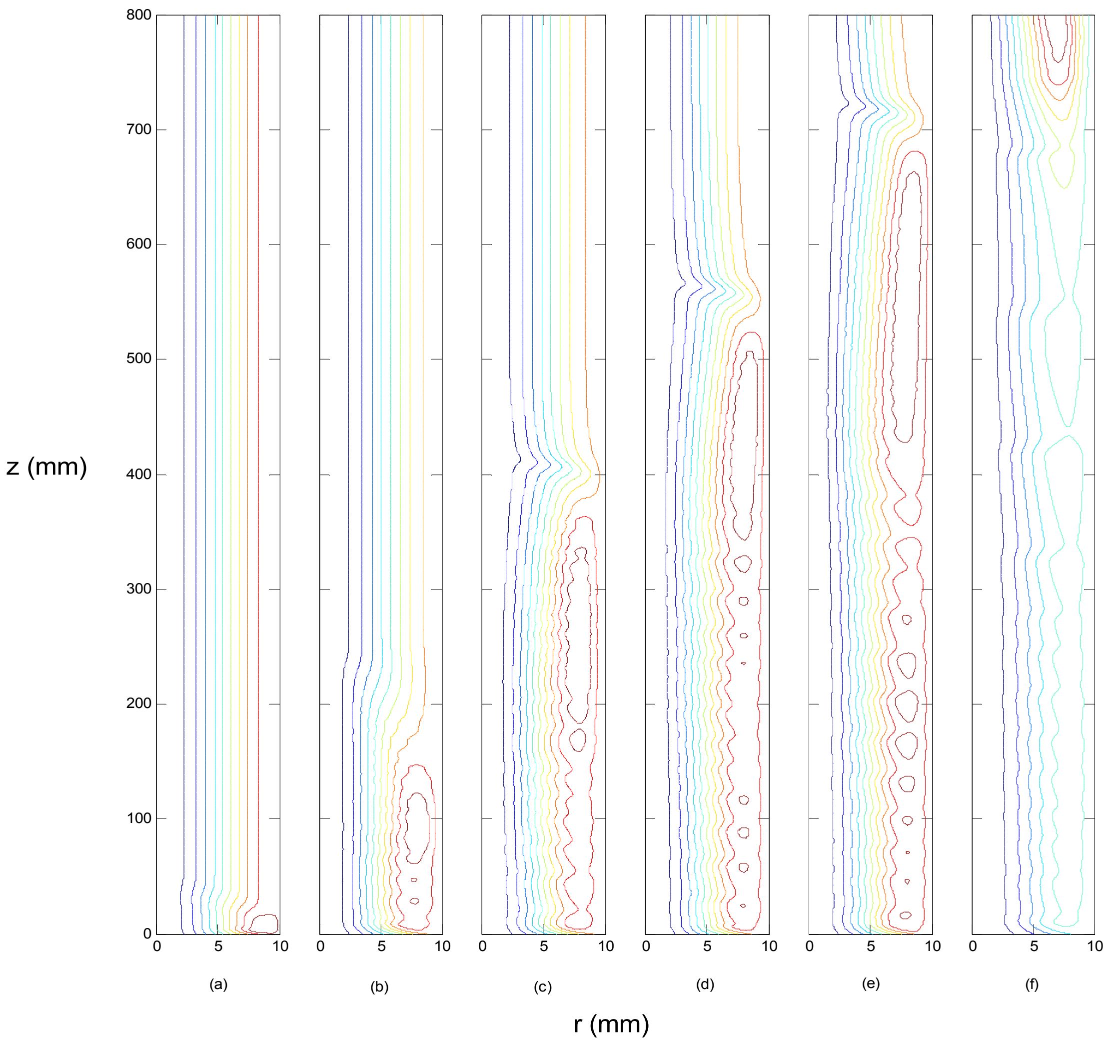

Fig.1: Time development of streamline along the tube for (a): 2 s, (b): 10 s, (c): 20 s, (d): 30 s, (e): 40 s and (f): 50 s.

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References

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