TRANSIENT MHD DOUBLE-DIFFUSIVE BY NATURAL CONVECTION OVER A VERTICAL SURFACE EMBEDDED IN A NON-DARCY POROUS MEDIUM

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The problem of transient, laminar, MHD double diffusive by free convection over a permeable vertical plate embedded in Darcy and non-Darcy porous medium is numerically investigated. Nonsimilarity solutions are obtained for constant wall temperature and concentration with a speciefed power law of mass flux parameter. The effects of magnetic parameter, inertial coefficient, Lewis number, buyancy ratio and lateral mass flux on heat and mass coefficients are presented and discussed.

INTRODUCTION

Recently, heat and mass transfer from different geometries embedded in fluid saturated porous medium has been studied extensively. This is due to the fact that these flows have many engineering and geophysical applications which include geothermal resources, building insulation, oil extraction, under ground disposal of nuclear waste, heat salt leaching in soils and many more. Heat and mass transfer by free convection in porous medium under boundary layer approximation has been studied by various researcher[1-3]. It is worth mentioning that all these studied were focused on steady state conditions and based on Darcy model. However, Darcy model is valid when the Reynolds number based on the pore size is less than unity. For fluids of high velocity and/or porous material of large pore radius, the Darcy model is inadequate since it neglects the porous medium inertia effect, which becomes significant. Inertial effects are taken into consideration in Forchheimer flow model which is a modification of the original Darcy law by adding the quadratic inertial term[4-6].

There has been a renewed interest in MHD flow and heat transfer in porous and clear domains due to the important effect of magnetic field on the performance of many systems using electrically conducting fluid such as MHD power generators and the cooling of nuclear reactors. In a review article[7], Ram presented an account of several steady MHD heat and mass transfer problems. Chamkha[8] considered the steady MHD free convection from vertical plate in a thermally stratified porous medium with Hall effect.

The aim of this work is to investigate transient MHD double diffusive of an electrically-conducting fluid by free convection over a flat plate embedded in Darcy and non-Darcy porous medium in the presence of surface suction or blowing and magnetic field effects.

Mathematical Analysis

Considering the unsteady non-Darcy MHD free double diffusive convection flow of an electrically-conducting fluid over an isothermal vertical porous plate embedded in a porous medium with suction or injection. Initially the wall is at constant temperature T_∞ and concentration C_∞, respectively. At $t = 0$, the wall temperature and concentration are suddenly raised to T_w and C_∞, higher than the ambient values T_∞ and C_∞, respectively and they maintained at these values for $t > 0$. A uniform magnetic field is applied normal to the plate. It is assumed that, the fluid properties are constant except the influence of density variation with temperature, which is considered only in the body force term. Both fluid and solid matrix are in local thermal equilibrium, the permeability porous medium is constant, viscous dissipation, Joule heating and thermal dispersion effects are negligible. Under these assumptions along with Boussinesq approximation, the governing equations are given by

$$\frac{\partial u}{\partial x}+\frac{\partial \mathrm{v}}{\partial y}=0 \tag{1}$$

$$\left(1+\frac{\sigma B_0{}^2 K}{\rho \nu}\right)\frac{\partial u}{\partial y}+\frac{C\sqrt{K}}{\nu}\frac{\partial u^2}{\partial y}=\left(\frac{Kg\beta_T}{\nu}\right)\frac{\partial T}{\partial y}+\left(\frac{Kg\beta_C}{\nu}\right)\frac{\partial C}{\partial y} \tag{2}$$

$$\frac{\partial T}{\partial t}+\mathrm{u}\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}=\alpha_e\ \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial t}+\mathrm{u}\frac{\partial C}{\partial x}+\mathrm{v}\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The physical problem suggests the following initial and boundary conditions

$$\left.\begin{array}{lll} u(x,y,0)=\mathrm{v}(x,y,0)=0.0, & & T(x,y,0)=T_\infty \\ \mathrm{v}(x,0,t)=-\mathrm{V_w}, & T(x,0,t)=T_w, & C(x,0,t)=C_w \\ u(x,\infty,t)=0, & T(x,\infty,t)=T_\infty, & C(x,\infty,t)=C_\infty \end{array}\right\} \tag{5}$$

where(x,y) are the dimensional distance along and normal to the plate, respectively, and (u, v) are the averaged velocity components along the x and y, directions respectively, t is the time ,T is the temperature, C is the concentration, β_T and β_T are the thermal expansion coefficient and the concentration expansion coefficient respectively, ν is the effective kinematic viscosity, α_e is the effective thermal diffusivity, F is the dimensional inertial coefficient, K is the permeability of the medium, respectively, $\mathrm{V_w}$ is the suction (>0) or injection (<0) velocity and D is the mass diffusion coefficient.

Introducing the following dimensionless variables and parameters

$$\left.\begin{array}{l} \eta=\frac{y}{x}\sqrt{Ra_x},\quad \psi=\alpha\sqrt{Ra_x}\,f(\eta,\tau),\quad \tau=\frac{\alpha\,Ra_x\,t}{x^2},\quad u=\frac{\partial\psi}{\partial y}\quad \mathrm{v}=-\frac{\partial\psi}{\partial x}, \\ \theta=\frac{T-T_\infty}{T_W-T_\infty},\quad \phi=\frac{C-C_\infty}{C_W-C_\infty},\quad Le=\frac{\alpha}{D},\quad Ha=\sqrt{\frac{\sigma B_0{}^2 K}{\rho\nu}},\quad N=\frac{\beta_C(C_W-C_\infty)}{\beta_T(T_W-T_\infty)}, \\ Ra_x=\frac{Kg\beta_T(T_W-T_\infty)x}{\alpha\nu},\quad \Gamma=\frac{F\sqrt{K}Kg\beta_T(T_W-T_\infty)}{\nu^2},\quad f_w=-\frac{2x\,\mathrm{v}_w(x)}{\alpha\sqrt{Ra_x}} \end{array}\right\}$$

where ψ is the stream function, τ is the dimensionless time, Γ is the Forchheimer number, Ha is the Hartmann number, Le is the Lewis number, f_w is the mass flux parameter. f_w is varied from –1 to 1. It is obvious that $f_w=0$ corresponds to impermeable wall, $f_w > 0$ corresponds to suction and $f_w < 0$ corresponds to injection.

Using the above similarity transformation the governing equations are reduced to

$$(1+Ha^2)f''+2\Gamma f''f' \ =\theta'+N\phi' \tag{6}$$

$$\theta''=-\frac{1}{2}f\theta'+\frac{\partial\theta}{\partial\tau}+\tau\left[f'\frac{\partial\theta}{\partial\tau}-\theta'\frac{\partial f}{\partial\tau}\right] \tag{7}$$

$$\phi''=-\frac{1}{2}\mathrm{Le}\,f\phi'+\frac{\partial\phi}{\partial\tau}+\tau\left[f'\frac{\partial\phi}{\partial\tau}-\theta'\frac{\partial\phi}{\partial\tau}\right] \tag{8}$$

and the initial and boundary conditions are transformed to

$$
\left.\begin{array}{llll}
 & f(\eta,0)=\theta(\eta,0)=0.0 & & \\
at \;\; \eta=0, & f=f_w, & \theta=1, & \phi=1 \\
as \;\; \eta\to\infty, & f'=0, & \theta=0, & \phi=0
\end{array}\right\} \tag{9}
$$

The physical quantities of fundamental interest of heat and mass transfer study are the heat and mass coefficients in terms of Nusselt and Sherwood numbers, respectively. The dimensionless heat and mass transfer coefficients can be expressed as

$$
\frac{Nu}{\sqrt{Ra_x}}=-\theta'(0), \qquad \frac{Sh}{\sqrt{Ra_x}}=-\phi'(0) \tag{10}
$$

RESULTS AND DISCUSSION

The resulting ordinary differential equations together with their boundary conditions have been solved numerically using an implicit finite difference technique.

A parametric study is carried out to investigate the effects of all involved parameters on the transient velocity, temperature and concentration profiles as well as the transient local Nusselt and the local Sherwood number.

CONCLUSION

The effects of magnetic field and lateral mass flux on transient MHD double diffusive of an electrically-conducting fluid by free convection over a flat plate embedded in Darcy and non-Darcy porous medium are numerically investigated. It was found that the presence magnetic field lowers both the Nusselt and Sherwood numbers in Darcy as well as Forchheimer flow regimes. Increasing the buoyancy ratio N or the fluid suction increases both Nusselt number and Sherwood numbers. The inertial effects reduce the heat and mass transfer coefficients. The Sherwood number Le is increased with Lewis number up to a limit, after which it decreases, with further increase in Le. This limit depends on the buoyancy ratio and the inertial parameter. Both the heat and the mass transfer rates tend to decrease as the time elapses.

REFERENCES

1. Singh, P. and Queeny, Free convection heat and mass transfer along a vertical porous medium, *Acta Mech*., Vol. 123, pp 69-73, 1997.
2. Nakayama, A. and Hossain, M. A., An integral treatment for combined heat and mass transfer by natural convection in porous medium, *Int. J. Heat Mass Transfer*, Vol. 38, pp 761-765, 1995.
3. Lai, F. C. and Kulacki, F. A., Coupled heat and mass transfer by natural convection from a vertical surface in porous medium, *Int. J. Heat Mass Transfer*, Vol. 34, pp 1189-1194, 1991.
4. Vafai, K. and Tien, C. L., Boundary and inertia effect on flow and mass in porous medium, *Int. J. Heat Mass Transfer*, Vol. 24, pp 1189-1194, 1982.
5. Whitacker S., The Forchheimer equation: a theoretical development, *Transport in porous medium*, Vol. 25, pp 201-211, 1996.
6. Manole, D. M. and Lage, J. L., The inertia effect on natural convection within a fluid saturated porous medium, *Int. J. Heat Fluid Flow*, Vol. 14, pp 376-384, 1993.
7. Ram, P. C., Recent developments of heat and mass transfer in hydro-magnetic flow, *Int. J. Energy Res.*, Vol. 15, pp 691-713, 1991.
8. Chamkha, A. J., MHD-free convection from a vertical plate embedded in a thermally stratified porous medium with hall effects, *Appl. Math. Modelling*, Vol. 21, pp 603-609,1997.