INTERNAL TRANSIENT FORCED CONVECTION WITH AXIAL DIFFUSION: COMPARISON OF SOLUTIONS VIA INTEGRAL RANSFORMS

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In the present work, transient laminar convection within a parallel-plates channel is studied through the use of the Generalized Integral Transform Technique (GITT) [1]. This hybrid numerical-analytical approach, especially useful in the development of benchmark solutions, is here analyzed in handling a test case [2-4] in transient convection within a wide range of diffusion and convection relative influences. Three different solution strategies, by varying the type of analytical filtering, are undertaken to allow for a critical comparison on their relative efficiency.

The developing thermal problem is solved for fully developed flow situation, considering axial diffusion in the energy equation [2-4]. An analysis is performed on the variation of Peclet number, so as to investigate the importance of the axial heat diffusion. Initially, the developed code employs a filtering solution for the steady-state and pure diffusion problem, but allows the reproduction of the infinite Peclet number results available in the literature (boundary layer formulation).

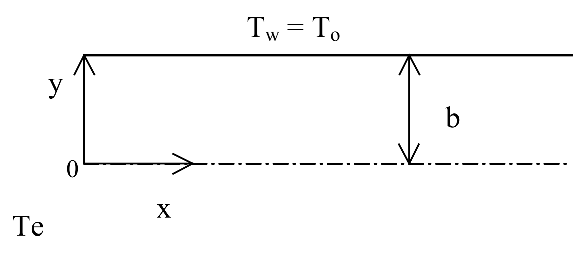

Figure 1 – Coordinates system and problem geometry

Considering the following dimensionless variables:

$$x = \frac{x^*/b}{\mathrm{Re}\,\mathrm{Pr}} = \frac{x^*}{b\,Pe};\; y = \frac{y^*}{b};\; u = \frac{u^*}{16 u_{av}};\; t = \frac{\alpha\, t^*}{b^2};\; L = \frac{L^*/b}{\mathrm{Re}\,\mathrm{Pr}};$$

$$T = \frac{T^* - T_o}{T_e - T_o};\; \mathrm{Re} = \frac{u_{av}\, 4b}{\nu};\; \mathrm{Pr} = \frac{\nu}{\alpha};\; \mathrm{Pe} = \mathrm{Re}\,\mathrm{Pr} = \frac{u_{av}\, 4b}{\alpha} \qquad (1)$$

the problem under concern is formulated in dimensionless form as:

$$\frac{\partial T(x,y,t)}{\partial t} + u(y) . \frac{\partial T(x,y,t)}{\partial x} = \frac{\partial^2 T(x,y,t)}{\partial y^2} + \frac{1}{Pe^2} \frac{\partial^2 T(x,y,t)}{\partial x^2}$$

$$0 < y < 1 \quad , \quad x > 0 \quad , \quad t > 0 \tag{2.a}$$

$$T(x,y,0) = 0 \quad , \quad x \geq 0 \quad , \quad 0 \leq y \leq 1 \tag{2.b}$$

$$T(0,y,t) = 1 \quad , \quad t > 0 \quad , \quad 0 \leq y \leq 1 \tag{2.c}$$

$$\frac{\partial T(x,0,t)}{\partial y} = 0 \quad , \quad t > 0 \quad , \quad x \geq 0 \tag{2.d}$$

$$T(x,1,t) = 0 \quad , \quad t > 0 \quad , \quad x \geq 0 \tag{2.e}$$

$$T(L,y,t) = 0 \quad , \quad t > 0 \quad , \quad 0 \leq y \leq 1 \tag{2.f}$$

where,

$$u(y) = \frac{3}{32}\left(1 - y^2\right). \tag{2.g}$$

An approximate analytical solution is also obtained for the same problem, through a combination of the methods GITT/Laplace Transform [5], which is critically compared to the above mentioned exact solution. This approximate analytical solution is then employed as a transient convective-diffusive filter in the complete solution of the original problem, with very good results, but with an increased computational cost in comparison with the first alternative.

Aimed at reducing the computational cost, a Local-Instantaneous Filtering strategy [6] is applied to the original problem, with excellent results, and convergence is analyzed by increasing the number of terms in the related series and/or by increasing the number of intervals in the filter updating.

The analysis shows that one should not neglect, a priori, the axial diffusion in certain problems without a careful investigation that accounts for the boundary and inlet conditions employed in the formulation. It is also demonstrated that the use of local-instantaneous filters and dynamic reordering of the expansions is of crucial importance in the optimization of GITT solutions.

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