TRANSIENT LAMINAR MIXED CONVECTION FLOW IN A VERTICAL TUBE UNDER HIGH Gr NUMBER CONDITION

C. T. Nguyen, G. Roy, M. A. Landry and S. E. B. Maïga
Faculty of Engineering, Université de Moncton, Moncton, N.B., Canada E1A 3E9

Because of its wide applications in engineering, the laminar mixed convection flow has received a special attention from the researchers in the past decades. A partial review of the relevant works in the domain may be found in[1,2]. For the flow in a vertical tube in particular, the pioneers works by Hanratty and colleagues[3,4] have clearly shown that the flow appears to be highly unstable and its transition to an unstable 'non-laminar' state (but not necessarily turbulent) has been observed at a relatively low Reynolds number. It has been also observed that such a transition was associated with an inflexion point on the axial velocity profile (case of assisted buoyancy forces) and with a detachment of the boundary-layer (case of opposing buoyancy forces). For the latter in particular, the flow regimes taking into account the flow reversal have been experimentally[5,6] and numerically[7,8] established. It is important to note that the influence due to the flow reversal on the onset of the instabilities has not been very well understood[1]. A few stability analyses based on the linear perturbations method[9-11] have given interesting insight into the nature rather complex of the instabilities which may occur in a fully-developed laminar flow. Since these studies assume infinitesimal perturbations, the resulting results cannot reflect the thermal and hydrodynamic behaviors under real operational conditions where often, great external disturbances prevail.

In the present work, we are interested to study the structure of the hydrodynamic and thermal fields and the transient behaviors of such a flow with an emphasis on the occurrence of the flow reversal phenomenon as well as its transition to an unstable state.

The problem consists of the simultaneously developing upward flow of air inside a uniformly heated vertical tube that is submitted to a uniform but time-dependent heat flux at the tube wall. The flow is assumed transient, laminar and three-dimensional (i.e. no assumption regarding the flow symmetry). The fluid is viscous, incompressible with constant properties except for its density which appears in the buoyancy terms (the Boussinesq's assumptions). Both the compression work and the viscous dissipation are considered negligible in the energy equation. The fluid enters at uniform axial velocity and temperature at the tube entrance section. The usual non-slip conditions and a uniform heat flux condition are prescribed on the tube wall. The wall heat flux varies as an *a priori* known linear function of time. At the tube exit section, the so-called 'pressure outlet conditions' prevail with prescribed pressure and fluid temperature. One can establish that the problem under consideration here is governed a set of 3 dimensionless parameters, namely the Reynolds number Re, the Prandtl number Pr and the Grashof number Gr.

The governing equations (the mass conservation equation, the Navier- Stokes equations along R, θ and Z directions and the energy equation) constitute a non-linear and highly coupled system. This PDE system has been successfully solved by employing the powerful FLUENT code which is based on the control-volume-based technique. The power-law scheme was used throughout to compute the diffusive and convective fluxes of heat and momentum while a second order in time scheme has been employed for the temporal terms. Several non-uniform grids were thoroughly tested and the 25 X 32 X 90 - respectively 25, 32 and 90 nodes along R, θ (0 to 360°) and Z directions with grid points highly packed near the tube entrance, exit and wall - has been adopted. As initial conditions, we employed the velocities and thermal fields corresponding to an isothermal convection case (i.e. Gr = 0) and proceed, by increasing progressively the wall heat flux, to another level of Gr. At each of the level of Gr desired, the flow and the thermal field have been carefully scrutinized in order to ascertain their structures as well as their nature, either stable or unstable, especially for case under sufficiently high heat flux condition (i.e. high Grashof number) where the occurrence of the reversal phenomenon may be observed.

The computer code has been successfully validated by comparing the numerical outputs to either analytical and numerical results available in the literature for (i) 'pure' forced flow (i.e. Gr=0) and (ii) steady-state mixed convection flow in vertical tube[12]. The code was then used to perform calculations for the case under study here.

The results have been first obtained for the steady-state ascendant air flow cases with Reynolds number fixed to be 250 and Gr number varying from 10^4 to $1{,}75X10^6$. It has been observed that the buoyancy forces have important effects on both the temperature and axial velocity of the fluid, in particular in the region near the tube wall where the acceleration of the fluid appears obvious. The heat transfer has been found to increase with the augmentation of Gr. At Gr=$1{,}5 \text{ X } 10^6$, the flow reversal occurs on the tube centreline near the exit section. The presence of such a reversal flow has drastically modified the structure of the flow and the thermal field in that area. The extent as well as the effects of this reversal zone have been found to increase with the Gr number.

On the other hand, the transient behaviours of the thermal and flow fields have also been investigated. We have established, for several cases with different Gr numbers, the temporal development of the temperature and velocity fields during the whole heating duration. Through the use of the 'numerical perturbation technique' which consists of generating 'numerical noises' on either temperature and velocity fields throughout the domain, we may now ascertain that the mixed convection flow appears to be stable before Gr=$7{,}5 \text{ X } 10^5$. Beyond this value, the convergence becomes rather tedious and extremely difficult which may indicate some foresight of a certain transition to another (unstable ?) state. We are now undertaking extensive calculations on this interesting issue and hope that some significant results will be available at the time of the symposium.

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