RADIATIVE HEAT TRANSFER TO CHEMICALLY REACTING HYDRO-MAGNETIC FREE-CONVECTION FLOW WITH HALL CURRENTS.

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The problem of radiative heat transfer to chemically reacting hydro-magnetic free convection flow with Hall current has a lot of applications in astrophysical studies. Bestman and Adjepong (1988) described the effect of unsteady hydro-magnetic free convection flow with radiative heat transfer where they observed that for limiting cases analytical solutions can be obtained for the temperature and velocity fields in a three-dimensional magneto-hydrodynamic free-convection flow with radiative heat transfer, past an infinite moving plate, in a rotating compressible viscous and optically transparent medium. Alagoa and Bestman (1993) observed in their MHD study of free convection and Hall current effects on waves in a semi-infinite plasma the appearance of steady streaming at all times for the velocity, temperature and magnetic fields. Bestman and Mbelegogu (1992) studied the effect of Hall currents on hydro-magnetic slip flow of a radiating fluid where the slip was provoked primarily by high temperature rather than by low pressure, and they found that the temperature decreases with increases in the radiation parameter and so does the concentration.

In this study, as in Bestman (1988), we consider the simple chemical reactions involving the diffusion of such species as NO, NO_2 and O_2 + O, which may result in the depletion of the ozone layer, for instance. A good review of this type of study is given in Rath (2002). We consider the unsteady flow of an incompressible, viscous, electrically conduction fluid past an infinite vertical porous plate in the presence of an externally applied magnetic field where we invoke the optically thin gray gas approximation. In the model we assume the temperature is high enough for radiative heat transfer to be significant. We also assume the gas to be optically thin and near equilibrium so that the equation for the radiative flux as given by Cogley et al. (1968), can be invoked i e.

$$\frac{\partial q_y}{\partial y} = K(T - T_o) \ where \ K = 4\int_o^\infty (\alpha_\lambda \frac{\partial B}{\partial T})_o \, d\lambda \tag{1}$$

where α is the absorption coefficient, B is Planck's function, λ is the frequency and subscript o refers to equilibrium conditions. In the mass diffusion equation k_r is the chemical rate constant, while

$$f(T) = T^\eta \exp\left[-\frac{E}{k^* T}\right] \tag{2}$$

is the generalized Arrhenius function in which η is a parameter, E' is the activation energy and k* is the Boltzmann constant.

Under the usual electromagnetic and Boussinesq approximations, and suitable non-dimensionalization the proposed governing equations therefore are:

$$\frac{\partial v}{\partial y} = 0 \tag{3}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{M}{1+m^2}(u+mw) + G_r\theta + G_c c \tag{4}$$

$$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} - \frac{M}{1+m^2}(w-mu) \tag{5}$$

$$P_r\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial y}\right)\theta = \left(\frac{\partial^2}{\partial y^2} - N\right)\theta + N \tag{6}$$

$$S_c\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial y}\right)c = \left(\frac{\partial^2}{\partial y^2} - k_r^2\theta^\eta \exp\left[-\frac{E}{\theta}\right]\right)c \tag{7}$$

subject to the following initial and boundary conditions

$$\begin{aligned} &u = 0 = w,\ \theta = 0,\ c = 0 \quad \text{for all } y \text{ for} \quad t \le 0 \\ &u = e^{a t}\ \ w = 0,\ \theta = 1,\ c = 1 \quad \text{for } y = 0 \quad t > 0 \end{aligned} \tag{8}$$

Bestman and Adjepong (1988) have shown that problems of this nature can be solved without loss of generality by seeking a perturbative scheme for the dependent variables of the sort

$$q = q^{(0)}(y) + \varepsilon q^{(1)}(y,t) + ... \tag{9}$$

$$\theta = \theta^{(0)}(y) + \varepsilon \theta^{(1)}(y,t) + ... \tag{10}$$

$$c = c^{(0)}(y) + \varepsilon c^{(1)}(y,t) + ... \tag{11}$$

Hence when equations (9) – (11) are substituted into the governing equations the problem is split into the steady state and the transient. These linearized differential equations can now be solved numerically. Results obtained show that the temperature decreases with increase in the Prandtl number and Suction parameter. A similar flow behaviour is exhibited by the species concentration, i e. decrease in species concentration with increase in Schmidt number. The main focus of the study is the effect of radiation, as in re-entry, and the effect of chemical reaction, as in the depletion of the ozone layer.

Our preliminary result for the case E = 0 = η which will correspond to a quiet night time activity shows that the species concentration decreases with increasing values of k_r. We note that as k_r increases the depletion rate increases as observed in Bestman (1988). For the temperature we observe a decrease in temperature as the radiation parameter increases as is observed in astrophysical environments, Cookey and Tay (2002). Hall current effects increase the complex velocity especially near the plate.

References

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