Extinction and the Electromagnetic Optical Theorem

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Abstract

The energy flow caused by single particles is examined to form a physically based understanding for extinction and the electromagnetic optical theorem. The behavior of the energy flow is explained in terms of wave interference. The optical theorem is shown to be a relationship based on the relative phase between the wave incident on a particle and the wave scattered by it. The relative phase is determined by the refractive and absorptive properties of the particle and reveals how extinction depends on both scattering and absorption.

1 Introduction

The optical theorem has a long history; its occurrence in electromagnetic theory begins more than one hundred years ago and analogs of the theorem are found in quantum mechanical and acoustical scattering [1]. Over the years, many derivations and implementations of the theorem have been given [2]-[5]. However, we have found no previous work that provides an explicit interpretation of the theorem in a physical (not mathematical) context. This work examines the optical theorem in detail and achieves a physical understanding of its meaning based on simulations of the energy flow caused by specific single particles.

2 The Optical Theorem

Consider a single particle that is illuminated by a plane wave polarized along the x-axis and traveling in the $\hat{\mathbf{n}}^{inc}$ direction, see Fig. (1) and Eq. (2) below. By expanding this wave into counter-propagating spherical waves using Jones' lemma, one can obtain an expression for the extinction cross section C^{ext} of the particle as

$$C^{ext} = \frac{4\pi}{k\mathcal{E}_o^{inc}} \operatorname{Im}\{\hat{\mathbf{x}} \cdot \mathbf{E}_1^{sca}(\hat{\mathbf{n}}^{inc})\}\,. \tag{1}$$

This is the *electromagnetic optical theorem* [5].

To the unfamiliar investigator, Eq. (1) can appear mysterious. The optical theorem relates C^{ext} to the imaginary part of the scattering amplitude $\mathbf{E}_1^{sca}$ evaluated in only the forward direction $\hat{\mathbf{n}}^{inc}$. However, extinction is due to scattering and absorption [3]. Scattering involves all directions whereas absorption is often independent of direction, so why should the optical theorem depend on only the forward direction, and how is absorption involved?

3 Theoretical Considerations

Suppose that the particle is located at the origin of the Cartesian coordinate system and surrounded by vacuum, see Fig. (1). The particle is described by a complex-valued refractive index m. Let $\mathcal{S}$ and V denote the surface and interior volume of the particle, respectively. The fields of the incident plane wave are

$$\mathbf{E}^{inc}(\mathbf{r}) = \mathcal{E}_o^{inc}\exp(ikr\hat{\mathbf{r}} \cdot \hat{\mathbf{n}}^{inc})\,\hat{\mathbf{x}}\,, \qquad \mathbf{B}^{inc}(\mathbf{r}) = \frac{k}{\omega}\hat{\mathbf{n}}^{inc} \times \mathbf{E}^{inc}(\mathbf{r})\,. \tag{2}$$

The wave number of the incident wave is $k = 2\pi/\lambda$ where λ is the wavelength. A harmonic time dependence given by $\exp(-i\omega t)$, where ω is the angular frequency, is assumed for all field quantities. Surrounding the particle is a spherical surface $\mathcal{S}_l$ of radius R_l that is large enough that points on its surface are in the far-field of the particle. The intersection of $\mathcal{S}_l$ with the y-z plane forms the C_l contour, see Fig. (1). The wave scattered by the particle can be found by solving Maxwell's equations using the volume integral relation [5]. In the far-field of the particle, the scattered wave takes the form of an outward traveling spherical wave with an angular profile given by the scattering amplitude $\mathbf{E}_1^{sca}$.

The Poynting vector $\mathbf{S}$ describes the energy flow in an electromagnetic wave [6]. Because both the incident and scattered waves exist at the observation point $\mathbf{r}$, $\mathbf{S}$ factors into three distinct terms. One of these terms, the cross term $\mathbf{S}^{cross}$, involves the fields of *both* the incident and scattered waves whereas the other terms involve the fields of only *either* the incident or scattered waves. The integral of the flow of the time-averaged cross term $\langle\mathbf{S}^{cross}\rangle_t$ through $\mathcal{S}_l$ gives the extinction cross section C^{ext}, and, it is this integral that forms a starting point for the derivation of the optical theorem, Eq. (1) [5].

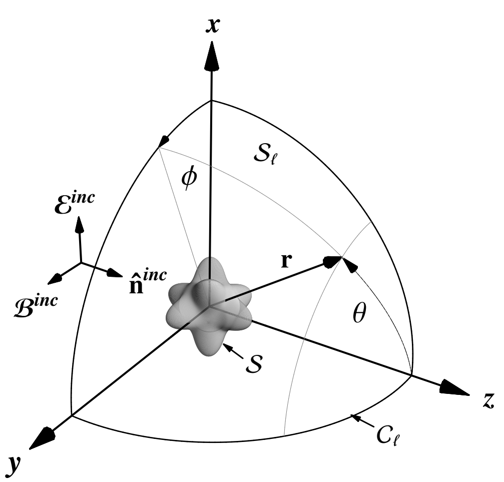

Figure 1: A scattering arrangement with a star-shaped particle at the origin. The incident wave is shown with the direction of its electric $\mathcal{E}^{inc}$ and magnetic $\mathcal{B}^{inc}$ fields pointing along the x and y-axes respectively and its propagation direction along $\hat{\mathbf{n}}^{inc}$. The observation point $\mathbf{r}$ is shown along with the surface $\mathcal{S}_l$ of radius R_l and the intersection of $\mathcal{S}_l$ with the y-z plane that forms the circular contour, C_l.

4 A Physical Picture of the Optical Theorem

Figure (2) reveals a major qualitative aspect of the behavior of the radial component of the $\langle\mathbf{S}^{cross}\rangle_t$ energy flow; the flow alternates from being inward to outward and does so more rapidly with direction as R_l increases. To see how integration of the $\langle\mathbf{S}^{cross}\rangle_t$ energy flow through $\mathcal{S}_l$ yields the extinction cross section, the integral $I^{cross}(\theta_s) = \int_{\partial\mathcal{S}_l}\langle\mathbf{S}^{cross}\rangle_t \cdot \hat{\mathbf{r}}\, \mathrm{d}a$ is also shown in Fig. (2). This integral is taken over the open surface $\partial\mathcal{S}_l$ which is formed by the part of $\mathcal{S}_l$ extending from $\theta = \pi$ to $\theta = \theta_s$. Because of energy conservation considerations, there is a negative sign in the relation $I^{cross}(\theta_s = 0) = -C^{ext}I^{inc}$, (where I^{inc} is the flux of the incident wave), which is why the curves in Fig. (2) for I^{ext} stop at the negative of $C^{ext}I^{inc}$ when $\theta_s = 0$.

By expressing the scattered wave as a spherical wave with an amplitude and phase shift given by the scattering amplitude $\mathbf{E}_1^{sca}$, we demonstrate that the optical theorem can be understood in a more 'physical' setting than is done in existing literature. The 'physical' understanding relies on the interpretation of extinction as being caused by the interference of the incident wave with the scattered wave. This interference causes the alternating energy flow shown in Fig. (2) and ultimately accounts for the value of C^{ext} via a phase shift relationship between the two waves. This phase depends, in part, on the wave inside of the particle. As the refractive ($\mathrm{Re}\, m$), or the absorptive ($\mathrm{Im}\, m$) properties of the particle vary, the internal wave changes which effects the phase of the scattered wave and hence forms the connection between extinction and the optical properties of the particle. In addition, the energy-flow interpretation of the optical theorem naturally

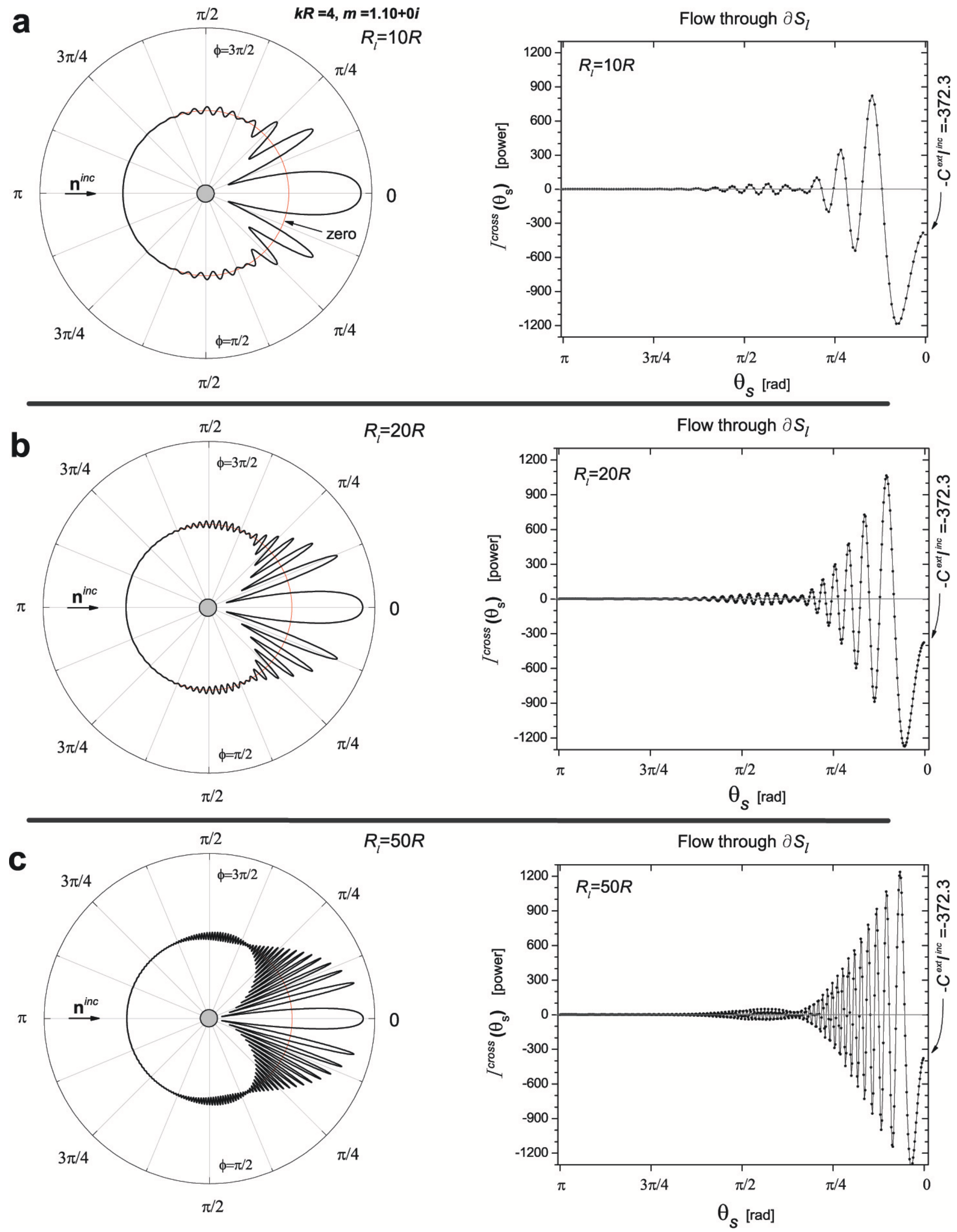

Figure 2: **Left column:** Polar plots of the radial component of the $\langle \mathbf{S}^{cross} \rangle_t$ energy flow through the C_l contour shown in Fig. (1). The particle is a sphere with a size parameter $kR = 4.08$ and refractive index $m = 1.10 + 0i$ and the scattered wave is calculated from Mie theory. The three polar plots labeled **a**, **b** and **c** show the energy flow for increasing contour radii $R_l = 10R$, $R_l = 20R$ and $R_l = 50R$, respectively. **Right column:** Plots of the integral I^{cross} for the contour radii corresponding to the matching polar plots in the left-hand column. The value of C^{ext} as calculated directly from Mie theory is indicated in the plots.

leads one to an understanding of the requirement regarding the size of a detector that is used to measure extinction, as discussed in Ref. [5].

Acknowledgments

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References

[1] Newton, R. G., 'Optical Theorem and Beyond,' Am. J. Phys. **44** (7), 639-642 (1976).

[2] van de Hulst, H. C., *Light Scattering by Small Particles*, (Dover, New York, 1981).

[3] Bohren, C. F., and Huffman, D. R., *Absorption and Scattering of Light by Small Particles* (Wiley, New York, 1983).

[4] Jackson, J. D., *Classical Electrodynamics* (Wiley, New York, 1999).

[5] Mishchenko, M. I., Travis, L. D. and Lacis A. A., *Scattering, Absorption, and Emission of Light by Small Particles* (Cambridge University Press, Cambridge, 2002).

[6] Rothwell, E. J. and Cloud, M. J., *Electromagnetics* (CRC Press, New York, 2001).