Optical trapping of nonspherical particles in the T-matrix formalism

Ferdinando Borghese,[1] Paolo Denti,[1] Rosalba Saija,[1] Maria Antonia Iatì,[1]
Onofrio Maragò[2]

[1] *Università di Messina, Dipartimento di Fisica della Materia e Tecnologie Fisiche Avanzate*
Salita Sperone 31, 98166 Messina, Italy
tel: +39-090-6765396, fax: +39-090-391382, e-mail: borghese@ortica.unime.it
[2] *Istituto per i Processi Chimico-Fisici del CNR, Via La Farina 237, 98122 Messina,Italy*
e-mail: marago@its.me.cnr.it

Abstract

We discuss the optical trapping of nonspherical particles in the transition matrix formalism. We also present some results of the application of the theory to trapping and manipulation of carbon nanotubes.

1 Introduction

The pioneering work of Ashkin et al. [1] has proved the practical feasibility of the trapping of small particles by highly focused laser beams. Actually, the configuration of the field focused by a high numerical aperture lens may be such that the resulting radiation force traps the particles within the focal region. The configuration of the field focused by an aplanatic lens has been described by Richards and Wolf [2] in terms of the rays that actually traverse the exit pupil. Although the Maxwell stress tensor enters the general expression of the radiation force [3], the literature reports several procedures to avoid using it when dealing with the optical trapping [4, 5].

In a recent paper the calculation of the trapping force on dielectric ellipsoids has been formulated in terms of the transition matrix approach [6] on the assumption that the incident field is a gaussian beam.

In this paper we reformulate the theory of optical trapping of nonspherical particles making full use of the Maxwell stress tensor and resorting to the transition matrix approach [7] in such a way that the final formulas are easily applicable to trapping of clusters of spherical scatterers. Actually, the radiation force that the field exerts on a particle is calculated by expanding the field in a series of spherical multipole fields and the field scattered by the particle is calculated through the transition matrix approach, which, in principle, does not require the particle to be spherical or small. Our aim is to apply the theory so formulated to the study of nanowires of silicon and nanotubes of carbon [8, 9] which present unique difficulties of trapping and manipulation.

2 Theory

The coordinate system that we adopt in our calculations is depicted in Fig. 1. The origin coincides with the focus of the lens whose optical axis coincides with the z axis. The particle under study lies at $\mathbf{R}_{O'}$ and the force exerted by the radiation is given by the equation

$$\mathbf{F}_{\mathrm{Rad}} = r'^2 \int_{\Omega'} \hat{\mathbf{r}}' \cdot \langle \mathsf{T}_{\mathrm{M}} \rangle \, \mathrm{d}\Omega' \,, \tag{1}$$

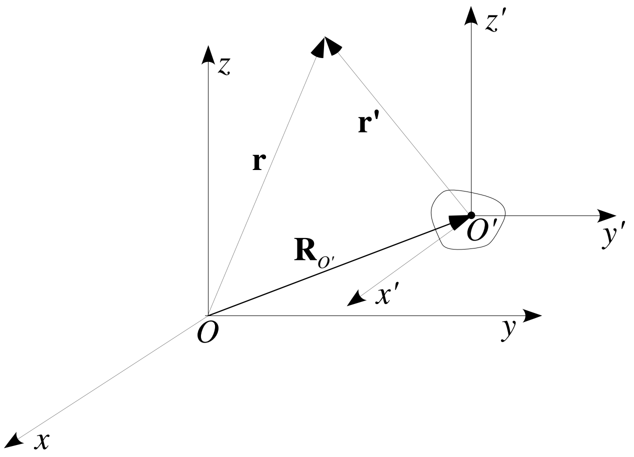

Figure 1: Coordinates

where the integration is over the full solid angle, r' is the radius of a large sphere S' with center O' at $\mathbf{R}_{O'}$ surrounding the particle, and $\langle \mathsf{T}_{\mathrm{M}} \rangle$ is the time averaged Maxwell stress tensor

$$\langle \mathsf{T}_{\mathrm{M}} \rangle = \frac{1}{8\pi}\mathrm{Re}\big[n^2\mathbf{E}' \otimes \mathbf{E}'^* + \mathbf{B}' \otimes \mathbf{B}'^* - \frac{1}{2}(n^2|\mathbf{E}'|^2 + |\mathbf{B}'|^2)\mathsf{I}\big] \ . \tag{2}$$

In Eq. (2), $\otimes$ denotes dyadic product, the asterisk indicates complex conjugation, the prime indicates that the fields are functions of $\mathbf{r}'$, and I is the unit dyadic. Of course, the fields that enter the definition of $\langle \mathsf{T}_{\mathrm{M}} \rangle$ are the superposition of the incident and of the scattered field. In our calculations we assumed that the incident field is a superposition of plane waves of the kind described by Richards and Wolf [2] and the scattered wave is calculated through the transition matrix of the particle. In this respect we stress that, in the calculation of the radiation force found convenient not to start from the formula obtained by Mishchenko [10] through the use of the optical theorem. We found more convenient to perform the integration in Eq. (1) using the asymptotic expansion of the multipole fields [11] up to terms of order $1/r$. In fact, higher order terms are easily seen to give a vanishing contribution to the radiation force. We cannot report here the mathematical details but a few points deserve a comment.

First we proved rigorously that the dyadic products $n^2\mathbf{E}' \otimes \mathbf{E}'^* + \mathbf{B}' \otimes \mathbf{B}'^*$ in Eq. (2) give no contribution to the radiation force not only when the incident field is a plane wave but also when it is the superposition of the plane waves, with different direction of propagation, that traverse the exit pupil of the lens.

Second, we were also able to demonstrate that, for the same superposition of plane waves, no contribution to the radiation force comes from the dot products $\mathbf{E}'_{\mathrm{I}} \cdot \mathbf{E}'^*_{\mathrm{I}}$ and $\mathbf{B}'_{\mathrm{I}} \cdot \mathbf{B}'^*_{\mathrm{I}}$.

Third, we used the fact that the incident field in the focal region can formally be written as a single plane wave whose amplitudes depend on the position of the particle. This allows us to calculate the scattered field through the usual transition matrix.

At present the work sketched here is still in progress and is carried on in collaboration with the experimental group at the Istituto per i Processi Chimico-Fisici of the CNR in Messina (Italy) and the group A. Ferrari at the University of Cambridge (UK).

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