Phase matrix for horizontally oriented ice crystals of cirrus clouds

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Abstract

The phase matrices for horizontally oriented ice plates of cirrus clouds are calculated within the framework of geometric optics. A method for retrieving aspect ratios of the plates by means of polarization measurements is discussed.

1 Introduction

The problem of light scattering by ice crystals of cirrus clouds is one of the current problems of the atmospheric optics. Optical properties of cirrus clouds are needed for incorporation in numerical models of radiative budget of the Earth and, consequently, in numerical models of weather forecasting and climate change. These optical properties have been calculating for last 20-30 years within the framework of geometric optics where the ice crystals were mainly assumed to be 3D randomly oriented (e.g. [1,2]). However, the ice crystals often reveal the tendency to be horizontally oriented because of aerodynamics laws. In particular, the horizontal orientation is manifested through numerous halo phenomena that are watched and classified for centuries [3]. As for quantitative data on the light scattering by preferably oriented ice crystals, they are rather poor [4-6]. The available data are represented as a number of figures that can be hardly used by other authors for some calculations. Moreover, a set of input parameters in these data is so small that they can be considered as preliminary or illustrative ones.

In this contribution, we focus on both the quantitative data and their physical interpretation. For brevity, this consideration is restricted by the most conventional and simple case of hexagonal ice plates that are horizontally oriented.

2 Reduced phase matrices

The conventional phase matrix $\mathbf{Z}$ (e.g. [7]) transforms the incident Stokes vector $\mathbf{I}_0 = (I_0, Q_0, U_0, V_0)$ into the Stokes vector $\mathbf{I} = (I, Q, U, V)$ of the scattered field

$$\mathbf{I}(\mathbf{n}) = \mathbf{Z}(\mathbf{n}, \mathbf{n}_0)\mathbf{I}_0(\mathbf{n}_0) \tag{1}$$

where $\mathbf{n}_0$ and $\mathbf{n}$ are the incident and scattering directions, respectively. This matrix is convenient for mathematical processing but the physical meaning of its elements is not intuitive. We prefer to use a reduced matrix $\mathbf{R}$ with more simple interpretation of its elements. Namely, let us consider the initial phase matrix $\mathbf{Z}$ as a set of four column vectors $\mathbf{Z} = (\mathbf{Z}_1. \mathbf{Z}_2, \mathbf{Z}_3. \mathbf{Z}_4,)$ where $\mathbf{Z}_1 = (Z_{11}, Z_{21}, Z_{31}, Z_{41},)$ and so on. The reduced matrix $\mathbf{R}$ is determined by the linear transformations of the column vectors as follows

$$\mathbf{R}_1 = \mathbf{Z}_1; \quad \mathbf{R}_2 = \mathbf{Z}_2 + \mathbf{Z}_1; \quad \mathbf{R}_3 = \mathbf{Z}_3 + \mathbf{Z}_1; \quad \mathbf{R}_4 = \mathbf{Z}_4 + \mathbf{Z}_1 \tag{2}$$

Then the column vectors $\mathbf{R}_j$ mean the Stokes vectors of the scattered field for different states of polarization of the incident light. They correspond to nonpolarized ($Q_0 = U_0 = V_0 = 0$), linearly polarized at 0º ($Q_0 =1$, $U_0 = V_0 = 0$), linearly polarized at 45º ($U_0 =1$, $Q_0 = V_0 = 0$), and circularly polarized ($V_0 =1$, $Q_0 = U_0 = 0$) incident light, respectively. Thus, all elements of the matrix $\mathbf{R}$ mean the results of the obvious experimental measurements. Here the first row means the intensities of the scattered light in the proper experiments. We shall normalize the other elements of a column vector to these intensities resulting in the degree of polarization of the scattered light for every polarization state of the incident light. This normalized matrix **N** with the elements

$$N_{1j} = R_{1j}\ ; \qquad N_{ij} = R_{ij} / R_{1j} \quad \text{for } i = 2, 3, 4 \tag{3}$$

is the goal of our numerical calculations. For the case of horizontally oriented crystals, the conventional spherical coordinate system is assumed where the azimuth angle θ is accounted from the vertical downward direction. The azimuth angle φ is accounted from an arbitrary chosen zero meridian in the direction of the unit vector $\mathbf{e}_\varphi$ that is determined by means of three right-handed basic unit vectors $\mathbf{n} = \mathbf{e}_\theta \times \mathbf{e}_\varphi$.

3. The reduced phase matrix N for horizontally oriented hexagonal ice plates

3.1 Phase functions

In a recent paper [8], we studied the phase functions of the horizontally oriented hexagonal plates in details. As known, the phase functions are concentrated along four horizontal circles. Two of them (the parhelic and subparhelic circles) reveal four sharp angular peaks. They are the forward peak, sundog, parhelion 120º, and peak 150º. As an example, a typical phase function is shown in Fig.1.

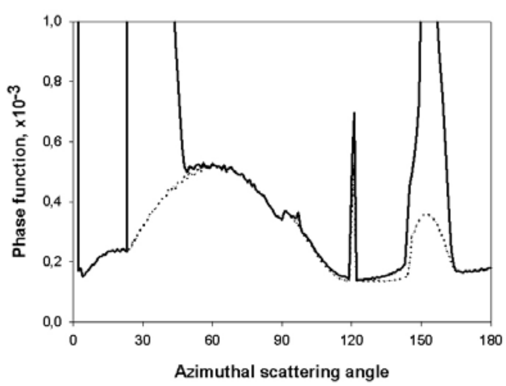

Fig.1. The phase function in the parhelic circle for a horizontally oriented hexagonal ice plate of the aspect ratio 0.2 at the incident angle of 75º. The dotted line corresponds to the residue after a subtraction of the abovementioned four peaks.

3.2 Degrees of polarization

Because of great intensity, these four peaks are easy measured in experiments. Hence, they are the most promising values for the inverse scattering problems, i.e. for retrieving crystal shapes and orientations from optical measurements. Polarization measurements inside these peaks should bring valuable additional information in the inverse problems since the polarization measurements are relational, i.e. they do not need some absolute detector calibration. So, the phase matrix calculated is a promising instrument for the scattering inverse problems.

We note that the parhelic circle is usually watched by an observer on the Earth when the sun radiation propagates through cirrus clouds. In this case, it is natural to assume that the incident light is nonpolarized, i.e. only the first column of the phase matrix is of practical interest. The subparhelic circle, on the contrary, can be watched from the Earth when a light source is located on the ground. This light source can be either a nonpolarized floodlight beam or a polarized laser beam. In the latter case, all columns of the phase matrix are of practical importance.

Let us consider the brightest peaks, i.e. the forward scattering peaks and sundogs. Table 1 and Fig.2 show the phase matrix **N** of Eq. (3) obtained for certain typical situations

Table 1: The normalized reduced matrix **N** of Eq. (3) in the forward scattering peak for the hexagonal plate of the aspect ratio of 0.1. The 2-4th rows are given in percents.

0.9	0.9	0.9	0.9	0.75	0.79	0.75	0.75	0.45	0.53	0.45	0.45
<1	100	<1	<1	4.59	100	4.59	4.59	16.6	100	16.6	16.6
0	0	>99	0	0	0	99.8	0	0	0	94.7	-10.1
0	0	0	>99	0	0	0	99.8	0	0	10.1	94.7

Parhelic circle, the incident angle are equals 15º, 45º, and 75º, respectively

0.03	0.03	0.03	0.03	0.05	0.02	0.05	0.05	0.21	0.14	0.21	0.21
-10.6	100	-10.6	-10.6	-61.9	100	-61.9	-61.9	-35.9	100	-35.9	-35.9
0	0	-99.4	0	0	0	-56.3	-14.1	0	0	91.2	-3.0
0	0	0	-99.4	0	0	14.1	-56.3	0	0	3.0	91.2

Subparhelic circle, the incident angle are equals 15º, 45º, and 75º, respectively

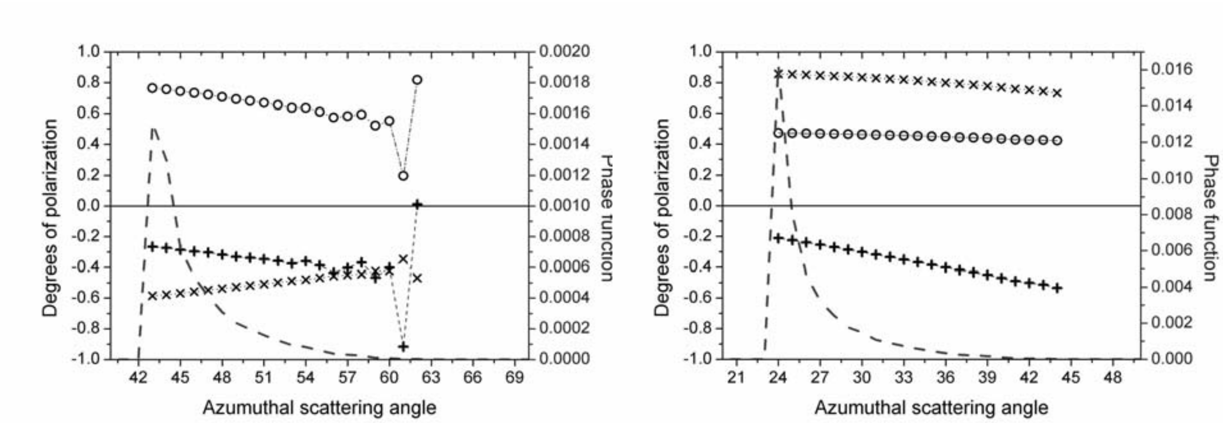

Fig.2. Polarization of sundog in the parhelic circle for the incident polarized light (i.e. the Stokes vector $N_{i3}(\varphi)$). The phase function N_{13} is shown by the dashed curves with the right ordinate axis. The left ordinate presents the other elements: N_{23} (+), N_{33} (×), and N_{43} (o). Aspect ratio is equal to 0.1. The incidence angles are 45 º (left) and 75 º (right)

Considering such kind of numerical data we conclude the following. The case of small incident angle (up to ≈ 30º) is not promising for retrieval of aspect ratios of the plates from the scattered radiation. For the large incident angles (> 30º), these are the elements N_{34} and N_{43} that prove to be sensitive to either the incident angles or the particle's aspect ratios. Moreover, their behaviors are rather regular. There is a simple physical explanation of this fact. Namely, the element N_{34} describes appearance of circular polarization in the case of incidence of linearly polarized light. A transformation of the linearly polarized light into circular polarization can be caused by only total internal reflections. So, the degree of circular polarization is an indicator for the number of the total internal reflections among the photon trajectories giving an essential contribution to the scattered light. The number of total internal reflections for the typical trajectories reflecting from the horizontal facets is a simple function of either the incident angle or particle's aspect ratio. Such physical regularities are true for other peaks, too, in the case of plates of arbitrary shape but of small aspect ratios. Therefore a method for diagnostics of aspect ratios of the plates by means of polarization measurements appears.

So, this is the bistatic lidar sounding of cirrus clouds that can be promising for diagnostics of horizontally oriented ice plates. In the bistatic lidar sounding scheme, not only the forward scattering peak, but other peaks, for example, sundogs can be used, too. In particular, Fig.2 demonstrates the strong dependence of the element N_{34} on the incidence angle within the sundog. Though our calculations are performed within the framework of geometric optics approach, they can be easy expanded to include diffraction. It is obvious that diffraction will not essentially distort the values describing degree of polarization. Therefore diffraction will not prevent to apply the method proposed for diagnostic of the aspect ratios of ice crystals in practice.

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