Scattering from a long helix

Ioseph Gurwich,[1] Moshe Kleiman,[2] and Nir Shiloah [2]

[1] *ELOP, P.O.Box 1166, Rehovot, 76110, ISRAEL*
[2] *Israel Institute for Biological Research, P.O.Box 19, Ness-Ziona, 74100, ISRAEL*
tel: + 972 8 938-1654, fax: + 972 8 938-1664, e-mail: moshekl@iibr.gov.il

Abstract

We consider here the electromagnetic wave scattering by a long and thin-wire (in comparison to the wavelength) helical particle. In contrast to several previous theoretical works, we adopt here the algorithm developed for scattering by a multi-layered fiber. In the present work a long helical particle is considered as a hollow cylinder with a thin non-homogeneous membrane for which the periodical boundary conditions are imposed.

1 Introduction

A helical particle is an exotic object, and till now it was scarcely considered in literature devoted to light scattering problems.
In rare works concerned with this problem[1, 2], numerical techniques are involved. In contrast to such approach, we develop here a formalism based on representation of a helical particle as thin non-homogeneous membrane and periodical boundary conditions. This allows for using the iterative technique and equations in the form obtained for a coated infinite cylinder[3] on each iteration step.

2 Basic considerations

Consider a helix oriented along z-axis (Fig. 1).

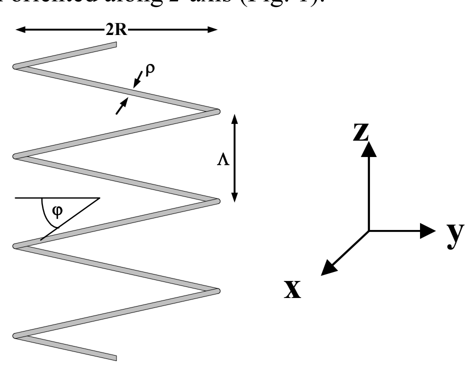

Figure 1: The geometry of helical particle

The equation of its central line is:

$$x_c = R_c \cos\varphi \\ y_c = R_c \sin\varphi \\ z_c = R\varphi \tag{1}$$

with $\varphi \in (-\infty,\infty)$, and the tangent unit vector $\hat{\mathbf{l}}$ is

$$\hat{\mathbf{l}} = \left(\frac{dx_c}{dl}, \frac{dy_c}{dl}, \frac{dz_c}{dl}\right) = \left(\frac{\cos\varphi}{\sqrt{1+h^2}}, -\frac{\sin\varphi}{\sqrt{1+h^2}}, \frac{h}{\sqrt{1+h^2}}\right) \tag{2}$$

where $h = \dfrac{\Lambda}{2\pi R_c}$ and l is the length parameter. Consider the attendant local coordinate (ξ,ψ,ζ),with $\hat{\zeta}$ coinciding with $\hat{\mathbf{l}}$. The outer surface points in these coordinates are $(\xi,\psi,0)$: $\xi^2+\psi^2=\rho^2$. However, for $\rho << \lambda$ (wavelength) we can accept that inside the helix (not in the volume of helix, but in the "wire" itself) the field is homogeneous relative to ξ,ψ coordinates, and thus the boundary conditions can be formulated on the central line: (x_c, y_c, z_c).

The case $R_c \to 0$ corresponds to an infinite thin wire, and the case $\eta = \rho/\Lambda \to \frac{1}{2}(0)$ corresponds to a hollow cylinder with a thin homogeneous membrane (absolutely transparent if η=0). In general, the helix can be considered as a hollow cylinder with a thin (non-homogeneous) membrane and periodical boundary condition (the following expression is not accurate, because of the round shape of a wire forming the helix, but for a thin wire we can ignore such an inaccuracy):

$$\varepsilon = \begin{cases} 1, z \in \left[\frac{\Lambda}{2\pi}\phi + n\Lambda + \rho, \frac{\Lambda}{2\pi}\phi + (n+1)\Lambda - \rho\right) \\ \varepsilon_h, z \in \left[\frac{\Lambda}{2\pi}\phi + n\Lambda - \rho, \frac{\Lambda}{2\pi}\phi + n\Lambda + \rho\right) \end{cases} \tag{3}$$

with ρ/Λ<<1, and n=…-1,0,1,... The proper Fourier series is

$$\varepsilon(z,\phi) = 1 + 2(\varepsilon_h - 1)\left\{\eta + \frac{1}{n\pi}\sum_{n=1}^{\infty}\sin(2\pi n\eta)\left[\cos(n\phi)\cos\left(2\pi n\frac{z}{\Lambda}\right) + \sin(n\phi)\sin\left(2\pi n\frac{z}{\Lambda}\right)\right]\right\} \tag{3-1}$$

and the similar series can be written for the refraction index m. Dealing with such a cylinder, we can formulate the periodical boundary conditions for z and τ components of **E** and **H**. Thus we have a hollow cylinder (Fig. 2 shows its cross-section), and three separated regions: 0- the inner medium (air), 1 – the helix, i.e., non-homogeneous membrane (gray area), and 2- the ambient medium (air).

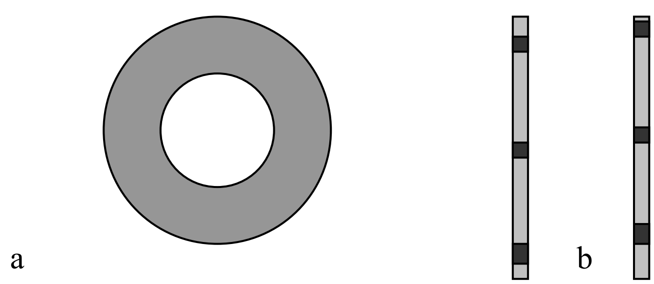

Figure 2: The hollow cylinder cross-section: a) upper view, b) side view

Here the inner radius is R_1=R_c-ρ, and the outer radius is R_2=R_c+ρ and ρ/R_c<< 1 is presumed.

3 Solution for scattered field

Strictly speaking the wave equation does not have a close solution for the present boundary condition. However, this can be shown that in cases where parameter $\eta = \dfrac{\pi\rho}{4\Lambda}$ is very small or close to 0.5: $\eta << 1$ or $|\eta - 0.5| << 1$, a convenient approximation does exist. In such an approximation one can represent the scattered field by the series of space (angular: θ will be the angle between z-axis and the scattering direction) Fourier harmonics.

It seems reasonable to assume that the scattered and inner fields have periodical dependence on the coordinate z with the space period Λ. Therefore we suppose the periodical dependence of scattering coefficients on z. In the present case the scattering coefficients (except of the incident $a_n^{(in)}$ field) have to be represented as the Fourier series. However, every Fourier term requires its own radial dependence; therefore, we have to write the solution in the form. In a certain approximation the fields in the j^{th} layer (j = 1,2) can be written as :

$$\begin{aligned}\mathbf{E}_j &= \sum_n \sum_l \left\{ Q_{n,l}^{(1)} \mathbf{N}_{n,j,l}^{(1)} + T_{n,l}^{(j)} \mathbf{M}_{n,j,l}^{(1)} - W_{n,l}^{(j)} \mathbf{N}_{n,j,l}^{(2)} - P_{n,l}^{(j)} \mathbf{M}_{n,j,l}^{(2)} \right\} e^{il\phi} e^{i2\pi lz/\Lambda} \\ \mathbf{H}_j &= -i \frac{km'_j}{\omega} \sum_n \sum_l \left\{ T_{n,l}^{(1)} \mathbf{N}_{n,j,l}^{(1)} + Q_{n,l}^{(j)} \mathbf{M}_{n,j,l}^{(1)} - P_{n,l}^{(j)} \mathbf{N}_{n,j,l}^{(2)} - W_{n,l}^{(j)} \mathbf{M}_{n,j,l}^{(2)} \right\} e^{il\phi} e^{i2\pi lz/\Lambda}\end{aligned} \tag{4}$$

Where **M** and **N** are cylindrical vector harmonics[4].

By writing m as $m(z,\phi) = \overline{m} + \delta m(z,\phi)$, where δm is represented approximately by a Fourier series analogous to (3-1) we write the boundary conditions for the inner and outer boundary in the form similar to that appearing for a case of a layered cylinder[3]. The solving procedure prescribes to use $m(z,\phi) = \overline{m}$ at the first step for getting the zero order space harmonic for the scattered field and the fields in the hollow cylinder layers. The scattering coefficients $\left\{ Q_{n,0}^{(j)}, T_{n,0}^{(j)}, W_{n,0}^{(j)}, P_{n,0}^{(j)} \right\}$ appear as the standard solution[3]. At the following stage one obtains higher order harmonics as perturbations with respect to the small parameter δm. The proper equations take the similar form, where the zero order field solution with the factor δm appears at the place of the incident field. Thus the similar procedure can be used in the iterative manner. This corresponds to the physical interpretation, where the mean-field generates higher order perturbations.

Since the helix is taken as infinitely long, the θ-directions can be found in the same manner as the diffraction angles for the infinitely long gratings. In case of a finite length helix each θ_l is replace by a (narrow) continuous spectral shape. Being interesting in the total energy scattered in a certain θ angular order, one can fulfill integration with respect to θ in the proper interval and then reduce formally the problem to the similar form for the mean value of the scattering coefficients, say $\left\langle Q_{n,l}^{(j)} \right\rangle_{|\theta \in \Delta\theta_l}$ instead of $Q_{n,l}^{(j)}(\theta \in \Delta\theta_l)$. In the case of the infinitely long helix, where a spectral function is reduced to the series of δ-functions we return to the original equations.

4 Extinction and scattering coefficients

Consider the general relations[4]:

$$\begin{aligned}\mathbf{W}_{sc} &= R \int dz \int d\phi (\mathbf{S}_{sc})_r \\ \mathbf{W}_{ext} &= R \int dz \int d\phi (\mathbf{S}_{ext})_r\end{aligned} \tag{5}$$

where R is the radius of a cylindrical surface around the helix (integration with respect to z can be fulfilled in the interval $[0,\Lambda]$) and

$$\begin{aligned} \mathbf{S}_{sc} &= \frac{1}{2}\operatorname{Re}\left\{\mathbf{E}_{sc} \times \mathbf{H}^{*}_{sc}\right\} \\ \mathbf{S}_{ext} &= \frac{1}{2}\operatorname{Re}\left\{\mathbf{E}_{in} \times \mathbf{H}^{*}_{sc} + \mathbf{E}_{sc} \times \mathbf{H}^{*}_{in}\right\} \end{aligned} \tag{6}$$

Since the incident field does not contain terms with factors $e^{ih\Lambda lz}$, coefficients $A^{(sc)}_{n,l}$, $B^{(sc)}_{n,l}$ with $l > 0$ will drop from expression for $\mathbf{W}_{sc}$. Thus in the common relations[4,5] we keep for Q_{ext} :

$$\begin{aligned} \operatorname{Re}(a_n) &= \operatorname{Re}\left\{A^{(sc)}_{n,0}\right\} \\ \operatorname{Re}(b_n) &= \operatorname{Re}\left\{B^{(sc)}_{n,0}\right\} \end{aligned} \tag{7-1}$$

but for Q_{sc} we have to take :

$$\begin{aligned} \left|a_n\right|^2 &= \sum_l \left|A^{(sc)}_{n,l}\right|^2 \\ \left|b_n\right|^2 &= \sum_l \left|B^{(sc)}_{n,l}\right|^2 \end{aligned} \tag{7-2}$$

5. CONCLUSION

In the present work we demonstrated a possibility of treating the problem of light scattering by a helical particle by using a Fourier approach. It is shown, that one can use a calculation procedure developed for a multilayered (hollow) cylinder to find all Fourier (diffraction) order of the scattered field. Thus a calculation technique turns out to be much simpler than it has been suggested before.

6. REFERENCES

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