Solving the diffraction problem of electromagnetic waves on objects with a complex geometry by the pattern equations method

Alexander G. Kyurkchan, Elena A. Skorodumova

Moscow Technical University of Communication and Informatics, Aviamotornaya Street 8a, 111024, Moscow, Russian Federation,
tel: +7 (495) 236-2267, e-mail: kyurkchan@yandex.ru

Abstract

The new method for solving the diffraction problem on objects with a complex geometry is offered. The problem is reduced to solving an algebraic system of equations with respect to the expansion coefficients of the scattering patterns by using of a series expansion of the scattering patterns in vector spherical harmonics. It's shown, that the method possesses high convergence rate. Examples of modeling of the scattering patterns of objects by the combination of last of objects of the more simple form (fragments of a complex objects) are considered. Reliability of the results obtained is validated by the using of the Optical theorem.

1 Introduction

The problem of effective modeling of the scattering characteristics of electromagnetic waves by objects with a complex geometry remains actual, because there are practically no effective methods of its solution. One of the most effective modern techniques for solving of a series of the diffraction problems is the pattern equations method (PEM). In particular, the high efficiency of this technique was demonstrated by solving of the diffraction problem for a group of bodies and for objects with a complex structure in an acoustical case [1]. One of important advantages of the PEM is its weak dependence of convergence rate of the computational algorithm on a distance between scatterers. In this paper, this technique is extended to an electromagnetic case.

2 Statement of the problem and it's solution

Consider the problem of waves scattering of the primary monochromatic electromagnetic field $\vec{E}^0$, $\vec{H}^0$ on a scattering objects with a complex geometry with latter to be presented as a combination of objects of more simple structure. Let's consider the case of two reflecting objects, for determinacy. It's possible to use this approach for any number of objects.

Let the impedance boundary conditions are set on surfaces S_j, $j=1,2$:

$$\left(\vec{n}_j\times\vec{E}\right)\Big|_{S_j}=Z_j\left[\vec{n}_j\times\left(\vec{n}_j\times\vec{H}\right)\right]\Big|_{S_j},$$

where Z_j – surface impedance, $\vec{n}_j$ – the unit normal vector to surface S_j, $\vec{E}=\vec{E}^0+\vec{E}_1^1+\vec{E}_2^1$, $\vec{H}=\vec{H}^0+\vec{H}_1^1+\vec{H}_2^1$ – the total field, $\vec{E}_j^1$, $\vec{H}_j^1$ – the secondary (diffraction) field, which satisfies to a homogeneous system of the Maxwell's equations everywhere outside of S_j ($\zeta=\sqrt{\mu/\varepsilon}$ – the wave impedance of a medium), and also to the Sommerfeld's condition on infinity

Let's take advantages by series expansion of the scattering patterns in vector spherical harmonics for reduction of an initial problem to a system of algebraic equations:

$$\vec{g}_j(\theta_j,\varphi_j)=\vec{F}_j^E(\theta_j,\varphi_j)=-\sum_{n=1}^{\infty}\sum_{m=-n}^{n}a_{nm}^j i^n\left(\vec{i}_r\times\vec{\Phi}_{nm}^j(\theta_j,\varphi_j)\right)-\sum_{n=1}^{\infty}\sum_{m=-n}^{n}b_{nm}^j i^n\zeta\,\vec{\Phi}_{nm}^j(\theta_j,\varphi_j),$$

where

$$\vec{\Phi}_{nm}^j(\theta_j,\varphi_j)=\vec{r}_j\times\nabla P_n^m(\cos\theta_j)\exp(im\varphi_j)$$

$P_n^m(\cos\theta_j)$ – associated Legendre functions [2], and functions $\vec{F}_j^E(\theta_j,\varphi_j)$ – are scattering patterns of electric field, satisfying in a so-called far zone (for $kr_j >> 1$) to asymptotical relations as:

$$\vec{E}_j^1 = \frac{\exp(-ikr_j)}{r_j}\vec{F}_j^E(\theta_j,\varphi_j) + O\left(\frac{1}{(kr_j)^2}\right).$$

Thus, our purpose is a deriving of an algebraic system for coefficients a_{nm}^j, b_{nm}^j, which can be expressed as following integrals:

$$a_{nm}^j = -\frac{\zeta}{4\pi}N_{nm}\int_{S_j}\left(\vec{n}_j'\times\vec{H}\right)\Big|_{S_j}\,\bar{\vec{e}}_{nm}^e(r_j',\theta_j',\varphi_j')\,ds_j', \quad (1)$$

$$b_{nm}^j = \frac{\zeta}{4\pi}N_{nm}\int_{S_j}\left(\vec{n}_j'\times\vec{H}\right)\Big|_{S_j}\,\bar{\vec{h}}_{nm}^e(r_j',\theta_j',\varphi_j')\,ds_j', \quad (2)$$

where the following labels are introduced:

$$N_{nm} = \frac{(2n+1)}{n(n+1)}\frac{(n-m)!}{(n+m)!},$$

$$\bar{\vec{e}}_{nm}^e(r_j,\theta_j,\varphi_j) = \nabla\times\nabla\times\left(\vec{r}_j\bar{\chi}_{nm}^j(r_j,\theta_j,\varphi_j)\right),\ \bar{\vec{h}}_{nm}^e(r_j,\theta_j,\varphi_j) = \frac{ik}{\zeta}\nabla\times\left(\vec{r}_j\bar{\chi}_{nm}^j(r_j,\theta_j,\varphi_j)\right),$$

$$\bar{\chi}_{nm}^j(r_j,\theta_j,\varphi_j) = j_n(kr_j)P_n^m(\cos\theta_j)\exp(-im\varphi_j),$$

$j_n(k\rho_j)$ – are the Bessel spherical functions of the first kind [2], line means a sign of complex conjugation.

As a result, with the using of expansions of the wave field:

$$\vec{E} = \vec{E}^0 + \sum_{\nu=1}^{\infty}\sum_{\mu=-\nu}^{\nu} a_{\nu\mu}^1\vec{H}_{\nu\mu}^{h1}(r_1,\theta_1,\varphi_1) - b_{\nu\mu}^1\vec{H}_{\nu\mu}^{e1}(r_1,\theta_1,\varphi_1) + a_{\nu\mu}^2\vec{H}_{\nu\mu}^{h2}(r_2,\theta_2,\varphi_2) - b_{\nu\mu}^2\vec{H}_{\nu\mu}^{e2}(r_2,\theta_2,\varphi_2),$$

$$\vec{H} = \vec{H}^0 + \sum_{\nu=1}^{\infty}\sum_{\mu=-\nu}^{\nu} a_{\nu\mu}^1\vec{H}_{\nu\mu}^{e1}(r_1,\theta_1,\varphi_1) + b_{\nu\mu}^1\vec{H}_{\nu\mu}^{h1}(r_1,\theta_1,\varphi_1) + a_{\nu\mu}^2\vec{H}_{\nu\mu}^{e2}(r_2,\theta_2,\varphi_2) + b_{\nu\mu}^2\vec{H}_{\nu\mu}^{h2}(r_2,\theta_2,\varphi_2),$$

where the wave spherical functions are expressed by the following relations:

$$\vec{H}_{nm}^{hj} = \nabla\times\nabla\times(\vec{r}_j\psi_n^m(\theta_j,\varphi_j)),\ \vec{H}_{nm}^{ej} = \frac{ik}{\zeta}\nabla\times(\vec{r}_j\psi_n^m(\theta_j,\varphi_j)),$$

$$\psi_n^m(\theta_j,\varphi_j) = h_n^{(2)}(kr_j)P_n^m(\cos\theta_j)\exp(im\varphi_j),$$

$$j = 1,\ 2,$$

$h_n^{(2)}(kr_j)$ – are the Hankel spherical functions of the second kind [2], and integral relations for coefficients a_{nm}^j, b_{nm}^j (1)-(2) we shall obtain the system of an algebraic equations of PEM:

$$\begin{cases} a_{nm}^j = a_{nm}^{j0} + \sum_{\nu=1}^{\infty}\sum_{\mu=-\nu}^{\nu}\left(G_{nm,\nu\mu}^{jj,aa}a_{\nu\mu}^j + G_{nm,\nu\mu}^{jj,ab}b_{\nu\mu}^j + G_{nm,\nu\mu}^{jl,aa}a_{\nu\mu}^l + G_{nm,\nu\mu}^{jl,ab}b_{\nu\mu}^l\right), \\ b_{nm}^j = b_{nm}^{j0} + \sum_{\nu=1}^{\infty}\sum_{\mu=-\nu}^{\nu}\left(G_{nm,\nu\mu}^{jj,ba}a_{\nu\mu}^j + G_{nm,\nu\mu}^{jj,bb}b_{\nu\mu}^j + G_{nm,\nu\mu}^{jl,ba}a_{\nu\mu}^l + G_{nm,\nu\mu}^{jl,bb}b_{\nu\mu}^l\right), \\ \qquad n = 1,2,...;\ |m| < n,\ j,l = 1,2,\ j \neq l. \end{cases}$$

All the coefficients in this system represent summation of two components: corresponding values when the impedance is equal to zero $Z_j = 0$, and additional addends, caused by difference of the value of impedance from zero.

3 An examination of the convergence of calculation algorithm

Our investigations has shown, that for the scatterers with an analytic boundary (spheres) four or five valid significant figures are already established when $N = 9 = 1.5\,kd_{1,2}$ even at the minimum distance between objects ($kr = 6.1$) (where d – is the maximum size of the scatterer). However in the case of bodies with nonanalytic boundary (cylinders) three or four valid significant figures are established only at $N = 2.1\ kd_{1,2}$.

These investigations has shown, that the convergence rate of the calculation algorithm remains almost the same high when scattering bodies are coming close together up to their contact, like in previously described acoustical case [1]. This fact allows us extend the PEM to solving the diffraction problem for scatterers with a complex geometry by their representation as a combination of objects of more simple form at minimum consumption of the computer resources.

4 Examination of mutual influence of objects

The difficulty of solving of the diffraction problem on a group of bodies consists in necessity to consider interaction of objects, which is related with rereflections between them. Fig. 1 illustrates dependence of integral scattering cross sections:

$$\sigma = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} |\vec{F}^E(\theta, \varphi)|^2 \sin\theta d\theta d\varphi$$

for two superellipsoids with $ka_{1,2} = 1$, $kc_{1,2} = 2$, $Z_{1,2} = 0$ on a distance between them, where $\vec{F}^E(\theta,\varphi)$ – is the scattering pattern for two bodies with (curve 1 and 2) and without taking into account the mutual influence, respectively. With increasing of the distance between objects the value of the common integral scattering cross section come close to summation of cross sections of separate bodies (curve 3). Besides, one can see on these figures, when bodies coming together there is a diminution of the aggregate cross section, calculated without taking into account the mutual influence. This can be explained by the "partial accumulation" of the power of an incident wave in the area between mirrors during the average period. Curve 4 corresponds to the integral cross section of superellipsoid of double size, to which the aggregate cross section of two bodies at their contact is coming close.

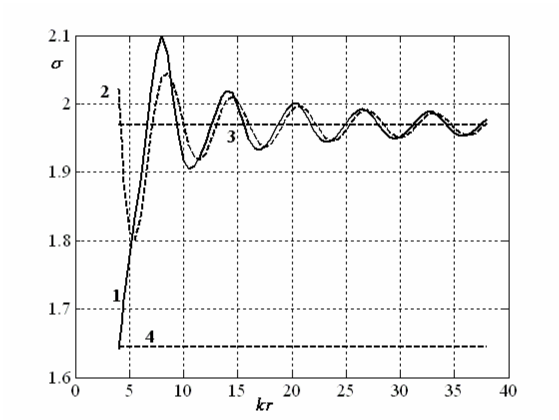

Figure 1.

5 Examination of possibility of the scattering characteristics modeling for bodies with a complex geometry

Let's carry out examination of the proposed method, based on comparison of scattering pattern of a single body with scattering pattern of the object, composed of halves of these bodies. Figures 2a and 2b show examples of such comparison for superellipsoids with $ka_{1,2} = 4$, $kc_{1,2} = 8$ for various values of the impedance. Fig. 2a corresponds to the value of the impedance equal to zero $Z_{1,2} = 0$, and fig. 2b – $Z_{1,2} = \zeta$, in both cases for the perpendicular incidence of wave. Figures show, that the differences of corresponding patterns are very small.

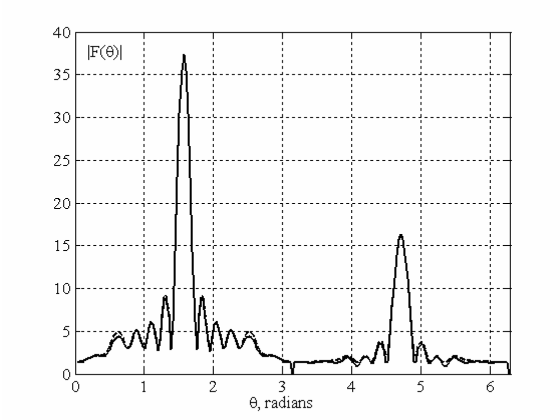

Figure 2a.

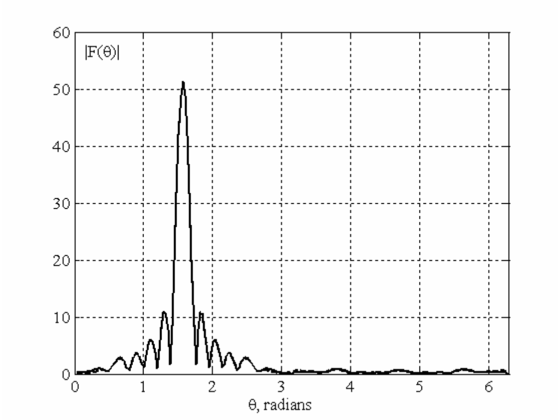

Figure 2b.

5 Verifying of the validity of the optical theorem

One of the methods of an estimation of the validity of the solution of the diffraction problem is verifying of fulfillment of the optical theorem, according to which [3]:

$$-\operatorname{Im}\left\{\vec{F}^{E}(\theta=\theta_0,\ \varphi=\varphi_0)\cdot\vec{p}\right\}=\frac{1}{2\lambda}\int_0^{2\pi}\int_0^{\pi}|\vec{F}^{E}(\theta,\ \varphi)|^2\sin\theta d\theta d\varphi,$$

where (θ_0,φ_0) – angles of incidence of a primary wave, $\vec{p}$ – a polarization vector, λ – a wave length.

Table 1. Verifying of the validity of the optical theorem for two objects at perpendicular incidence of a wave

Two spheres, $ka_{1,2}=3$			Two cylinders, $ka_{1,2}=3$, $kh_{1,2}=6$		
kr	σ	$-\operatorname{Im}\left\{F_\theta^E(\theta_0,\varphi_0)\right\}$	kr	σ	$-\operatorname{Im}\left\{F_\theta^E(\theta_0,\varphi_0)\right\}$
6.1	9,03726456130122	9,03726456130087	6.1	14.2118	14.3938
7	9,69071815232501	9,69071815232500	7	15.8345	15.6466
10	9,90163562078583	9,90163562078574	10	14.9745	14.9084

Table 1 shows, that for two spheres the accuracy of fulfillment of the optical theorem practically coincides with the machine precision of evaluations. However for cylinders the accuracy is noticeably lower, that can be explained by nonanalytic boundaries of scatterers, but also is quite acceptable.

Acknowledgements

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References

[1] A. G. Kyurkchan and E. A. Skorodoumova, “Modeling the characteristics of the waves scattering by a group of scatterers,” Journal of Quantitative Spectroscopy and Radiative Transfer. **100**, 207-219 (2006).

[2] N.Ya.Vilenkin, *Special Functions and Representations Group Theory*, (Nauka, Moscow, 1965 (in Russian)).

[3] C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley, New York, 1983).