Solution of wave diffraction problems by method of continued boundary conditions combined with pattern equation method

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Abstract

The integral-operator equation of the pattern equation method is deduced using a method of the continued boundary conditions. Generally speaking the deduced equation has the approximate character, however it is applicable for the solution of diffraction problems at smaller restrictions on scatterer geometry, than the rigorous equation. Numerical examples are considered.

1 Introduction

For almost 15 years the pattern equations method (PEM) has been successfully applied to the solution of the broad spectrum of wave diffraction and propagation problems. However, essential limitation of the method is that in it's strict formulation it is not applicable to the solution of diffraction problems on bodies with non analytical (in particular, piece-smooth) boundary caused by divergence of Sommerfeld-Weil integral representation in singular points of a wave field. The method of continued boundary conditions (MCBC) suggested recently allows to overcome this limitation. The trick is that according to MCBC, the boundary condition is satisfied not on boundary S of scatterer, but on some surface S_δ, covering S and separated from it by some sufficiently small distance δ. It leads to the approximate statement of a problem, however, as a result all difficulties related to singular points of a wave field on scatterer boundary in case it has breaks, corners, edges, etc., as well as difficulties related to singularity of the corresponding integral equation kernels are removed. Computational algorithm thus becomes significantly simpler and practically universal.

PEM integral-operator equation (in general approximate), which can be deduced using MCBC, is applicable under more general assumptions of scatterer geometry, than the exact equation of the method. For compactness we consider a diffraction problem on perfectly conducting scatterer. However the basic ideas of this approach are entirely extended to the vector problems.

2 Derivation of PEM integral-operator equation

It has been shown [1,2], that in framework MCBC the boundary problem can be reduced to the solution of Fredholm integral equation of the I[st], and II[nd] kind with smooth kernel. In particular, in case of perfectly conducting scatterer MCBC gives the following equation

$$\vec{J}_\delta = \vec{J}_\delta^0 - \left. \left(\vec{n} \times \frac{k}{4\pi} \int_S \left(\vec{J} \times \nabla G_0 \right) ds' \right) \right|_{S_\delta}, \qquad (1)$$

where $\vec{J} = (\vec{n} \times \vec{H})\big|_S$, $\vec{J}_\delta = (\vec{n} \times \vec{H})\big|_{S_\delta}$, $\vec{J}_\delta^0 = (\vec{n} \times \vec{H}^0)\big|_{S_\delta}$, and $\vec{H}^0$, $\vec{H}^1$, $\vec{H} = \vec{H}^0 + \vec{H}^1$ are primary, scattered and total magnetic field vectors respectively, $G_0 = \dfrac{\exp(-ikR)}{kR}$ is free-space Green function, $R = \left| \vec{r} - \vec{r}' \right|$.

By multiplying both parts of the Eq. (1) on $\exp\{ik\rho_\delta(\theta,\varphi)\cos\gamma\}$, $\cos\gamma = [\sin\alpha\sin\theta\cos(\beta-\varphi)+\cos\alpha\cos\theta]$, integrating on S_δ and taking advantage of generalized Sommerfeld-Weil representation for function G_0:

$$G_0 = \frac{1}{2\pi i}\int_0^{2\pi}\int_0^{\pi/2+i\infty} \exp(-ikr\cos\alpha + ikr'\cos\hat{\gamma})\sin\alpha d\alpha d\beta ,$$

were

$$\cos\hat{\gamma} = \sin\alpha\sin\hat{\theta}\cos(\beta-\hat{\varphi})+\cos\alpha\cos\hat{\theta},\ \cos\hat{\theta} = \sin\theta\sin\theta'\cos(\varphi-\varphi')+\cos\theta\cos\theta' ,$$

$$\sin\hat{\theta}\cos\hat{\varphi} = \sin\theta'\sin(\varphi'-\varphi),\ \sin\hat{\theta}\sin\hat{\varphi} = \sin\theta\cos\theta' - \cos\theta\sin\theta'\cos(\varphi-\varphi') ,$$

we obtain the following integral-operator equation of PEM relative to the scattering pattern $\vec{F}^E(\alpha,\beta)$

$$\vec{F}_\delta^E(\alpha,\beta) = \vec{F}_\delta^{0E}(\alpha,\beta) + \frac{k}{8\pi^2 i}\int_{S_\delta} e^{ik\rho_\delta(\theta,\varphi)\cos\gamma}\left\{\vec{n}\times\left[\nabla\times\int_0^{2\pi}\int_0^{\pi/2+i\infty} e^{-ikr\cos\alpha}\hat{\vec{F}}^E(\alpha',\beta';\theta,\varphi)\times \sin\alpha' d\alpha' d\beta'\right]\Big|_{S_\delta}\right\}ds. \quad (2)$$

Here $\vec{F}_\delta^E(\alpha,\beta) = \int_{S} \vec{J}_\delta(\vec{r})\exp[ik\rho_\delta(\theta,\varphi)\cos\gamma]ds$, $\hat{\vec{F}}^E(\alpha',\beta';\theta,\varphi) = \int_S \vec{J}(\vec{r}')\exp[ik\rho(\theta',\varphi')\cos\hat{\gamma}]ds'$ is the generalized scattering pattern [3].

Generally speaking the Eq. (2) is approximate, since at its derivation it was assumed, that $\vec{F}^E(\alpha,\beta) \cong \vec{F}_\delta^E(\alpha,\beta)$. However this equation is now applicable to the diffraction problems on bodies with non-analytical boundary. If boundary *S* is analytical the obtained equation becomes exact. It is interesting to note, that integral-operator equation of PEM cannot be derived from standard current Fredholm integral equation of a IInd kind even for bodies with analytical boundary because of the simple layer potential normal derivative jump.

If scatterer is weakly non-convex [3], it is more appropriate to use the equation Eq. (2) for solving diffraction problem, since the corresponding computation algorithm converges quite fast [3]. However, in case of strongly non-convex scatterers or thin screens, the Eq. (1), which is usually solved using local approximation of the sought current $\vec{J}(\vec{r}')$, is more suitable.

3 Numerical examples

The scattering pattern of plane electromagnetic wave propagating at angles $\varphi_0 = 0$, $\theta_0 = 0$ incident on a circular cylinder with a radius *ka*=3 and height *kh*=10 was calculate using the Eqs. (1) and (2). Results of calculations have graphically coincided. The maximal number *N* of the spherical harmonic used at unknown scattering pattern approximation in the Eq. (2) has been set 15, and number of basic functions *M*, used for approximation of unknown current in the Eq. (1) was set 128. The accuracy of the results obtained with Eq. (1) was evaluated by the residual of the boundary condition, calculated in points between collocation points. This residual is shown on figure. 1.

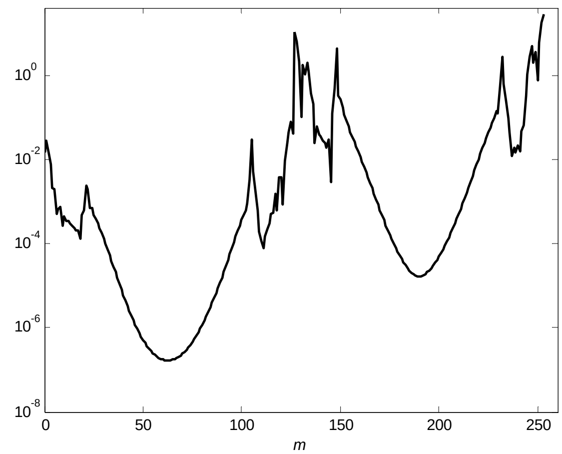

Figure 1: The residual for a circular cylinder with a radius *ka*=3 and height *kh*=10.

Thus, for bodies which geometry allows Eq. (2) for solving diffraction problem, the later is more suitable. However, as mentioned above, using approximation Eq. (3) for reducing Eq. (2) to algebraic system to solve, for example, diffraction problem on thin screen, is not acceptable. If we approximate the pattern $\vec{F}^E(\alpha,\beta)$ in Eq. (2) by sum

$$\vec{F}^E(\alpha,\beta)=\sum_{n=1}^{N}\vec{J}(\vec{r}_n)\exp[ik\rho_n(\theta,\varphi)\cos\gamma],$$

where $\vec{r}_n$ are the position vectors of the points Ω_n, situated everywhere dense on *S*, i.e. $\overline{\{\Omega_n\}_{n=0}^{\infty}}=S$, Eq. (2) leads to algebraic system with ill-conditioned matrix. In such situation using Eq. (1) is more expedient for solution boundary problem.

Let's consider now a diffraction problem, for which Eq. (2) is inapplicable. Solution of diffraction problem for plane wave at $\theta_0=0$ was obtained on a parabolic mirror defined as $\rho(\theta)=f/\cos^2(\theta/2)$, where *f* is the focal length, *kf*=20. Figures 2 and 3 show the scattering pattern in the plane $\varphi=[0,\pi]$ for F_θ^E (solid) and in the plane $\varphi=[\pi/2,3\pi/2]$ for F_φ^E (dashed) and the residual of the boundary condition, calculated in points between collocation points, respectively, obtained at *M*=64, *Q*=1, $\vec{p}=\vec{i}_x$. It can be seen that the solution has the acceptable accuracy.

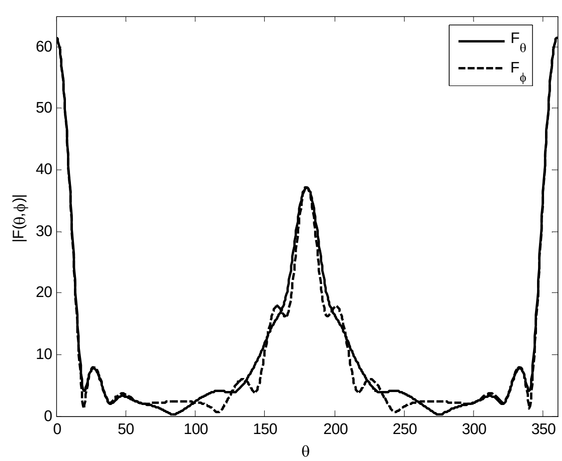

Figure 2: The scattering pattern for a parabolic mirror with *kf*=20.

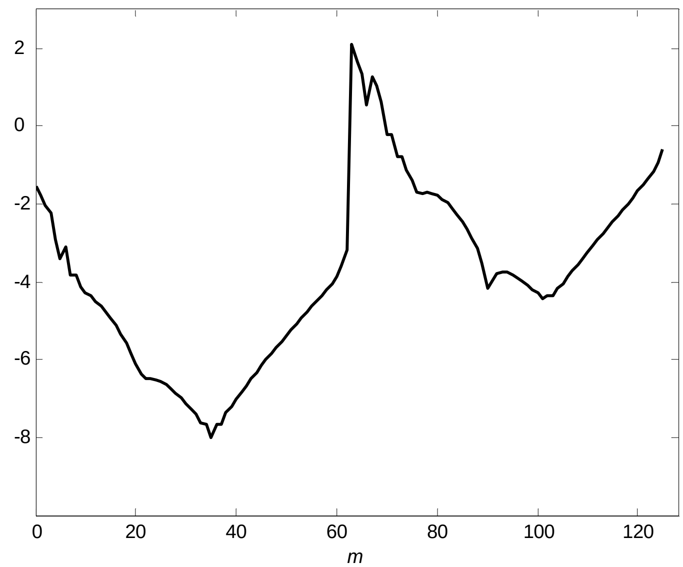

Figure 3: The residual for a parabolic mirror with *kf*=20.

4 Conclusion

Thus, MCBC allows the reduction of a boundary value problem not only to the integral equations with a smooth kernel with respect to sources density on scatterer surface, but also to equations with respect to a scattering pattern of a body, i.e. to a field characteristic in a long-distance zone. This gives the reason to think MCBC one of the most universal method for solving diffraction problems.

Acknowledgments

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