# **Coherence effects in systems of Dipolar Bi-Spheres**

O. Merchiers<sup>1</sup>, F. Moreno<sup>1</sup>, J.M. Saiz<sup>1</sup> and F. González<sup>1</sup>

<sup>1</sup> Universidad de Cantabria, Grupo de Óptica, Departamento de Física Aplicada, Avda de los Castros, 39005 Santander, Spain tel: +34 942 201868, fax: +34 942 201402, e-mail: olivier.merchiers@unican.es

#### Abstract

We study the behavior of the Negative Polarization Branch (NPB), using a scattering system which consists of two dipolar scatterers separated by a fixed distance and freely floating in space. For such a system, a resonance spectrum is obtained if one plots the scattering cross-section as a function of the polarizability. The excited resonances correspond with specific oscillation modes of the electric (and magnetic) dipole moments and arise due to the interaction between the dipoles. We show that the NPB can be generated if the system is put into the right resonance mode, but can also be suppressed if placed in a so called longitudinal mode. The effect of a magnetic permeability different from one on the NPB will also be considered.

# **1** Introduction

Coherence effects in or close to the backscattering direction have been the focus of attention of many researchers involved in the theoretical and experimental analysis of the propagation of electromagnetic waves in random dense media where multiple scattering is important. On one hand we have the enhancement of the scattered intensity in the backward direction (EBS) and on the other hand we have the polarimetric opposition effects. Based on observations, two types of opposition effects are usually distinguished. The first one, often called the polarization opposition effect (POE) which appears as a narrow asymmetric branch and the second one, often referred to as negative polarization branch (NPB) and appears as a wide symmetric branch around the backward direction in the linear polarization coefficient (LPC) [1,2]. It is considered that the coherent backscattering mechanism is responsible for the EBS, the POE and the NPB [3]. However, it is assumed that the coherent backscattering is not the only contribution to the NPB. These phenomena have been observed in experiments related to the scattering of unpolarized electromagnetic radiation by astronomical objects [1] and have also been reproduced in laboratory experiments with both volume and surface geometries [2,3].

In this work we present a study of the previously cited NPB using a model constituted by two dipoles separated by a fixed distance. We will call this system a Dipolar Bi-sphere (DBS). This simple model has already been introduced by other authors to study coupling resonances [4] and the EBS [5]. The aim of this contribution is to study the effect of coupling resonances of the DBS on the coherence effects [6], for both non-magnetic ( $\mu = 1$ ) and magnetic particles ( $\mu \neq 1$ ). The reason why we introduce the magnetic permeability in our study is because of the high interest for metamaterials during the last six years. While only very recently those materials have been seen to operate in the visible region [7] and only for slab geometries, an early theoretical study by Kerker et al. [8] showed that small spheres with  $\mu \neq 1$  have some very interesting scattering properties. Herein lays the interest of studying scattering systems with magnetic properties.

# 2. Theory

To obtain the scattered electromagnetic field from our system, we used a generalized version of the coupled dipole method [9] which was introduced by Mulholland et al. in [10]. This generalized method

enables us to compute the scattered electric field when the scatterers have a magnetic permeability different from 1. However, the method can equally well be applied for particles with  $\mu = 1$ . The method consists in writing each electric (**d**<sub>i</sub>) and magnetic dipole moment (**m**<sub>i</sub>) as a contribution of the incident field and the dipole moments induced by the rest of the particles. One has then to solve the system of 2N linear vector equations in **d**<sub>i</sub> and **m**<sub>i</sub> given by

$$\mathbf{d}_{i} = \boldsymbol{\alpha}^{e}_{i} \mathbf{E}^{o}_{i} + \sum_{i \neq j}^{N} \boldsymbol{\alpha}^{e}_{j} \left[ \mathbf{C}_{ij} \mathbf{d}_{j} - \sqrt{\frac{\varepsilon_{o}}{\mu_{o}}} \mathbf{G}_{ij} \mathbf{m}_{j} \right]$$

$$\mathbf{m}_{i} = \boldsymbol{\alpha}^{m}_{i} \mathbf{H}^{o}_{i} + \sum_{i \neq j}^{N} \boldsymbol{\alpha}^{m}_{j} \left[ \mathbf{C}_{ij} \mathbf{m}_{j} + \sqrt{\frac{\mu_{o}}{\varepsilon_{o}}} \mathbf{G}_{ij} \mathbf{d}_{j} \right]$$
(1)

where  $C_{ij}$  and  $G_{ij}$  are the interaction matrices and  $\alpha_j^e$  and  $\alpha_j^h$  are the electric and magnetic polarizabilities respectively. Once the electric and magnetic dipole moments are obtained by matrix inversion, the scattered electric field is easily obtained.

The calculations are made for  $N_T$  random orientations of the scattering system, we then obtain an averaged value of scattering cross-section  $\sigma_S$ .

### 3. Results

#### 3.1 Particles with $\mu = 1$

In figure 1(a) we show the averaged scattering cross section for  $\varepsilon = -2.013$ ,  $\mu = 1$  and P-polarized incident light versus the interparticle distance r between the particles. The results for the S-polarization are identical due to symmetry. As can be seen in figure 1a, successive resonances are excited. Each one of these resonances correspond to a specific oscillation mode of the dipoles.



Figure 1: (a) Scattering cross-section  $\sigma_S$  as a function of the interparticle distance for  $\varepsilon = -2.013$ ,  $\mu = 1$ . The arrows show the oscillation mode for each dipole when placed in the corresponding resonance. (b) LPC<sub>min</sub> and |LPC|<sub>max</sub> obtained for totaly unpolarized incident radiation.

Figure 1(b) represents two aspects of the linear polarization coefficient (LPC). LPC min represents the minimum of the LPC in the interval of scattering angles from 0° to 180° and |LPC| max is the maximum of the absolute value of the LPC in the same interval. We see that the peaks (2), (3) and (4) of figure 1(a) correspond to minima in the LPCmin. This means that the resonances (2), (3) and (4) produce NPB. This is not the case for resonance (1), which instead creates a minimum in the plot of |LPC| max vs r. From figure 1, we could thus deduce that for an interparticle distance  $r = \lambda$ , the produced NPB is entirely generated by the contribution of mode (2). However, an other analysis based on the eigenvector decomposition of the local dipole moments (see equation (2)) shows that the anti-symmetrical state does also contribute to the NBP.

$$\left|d\right\rangle = \sum_{n=1}^{3N} \frac{\left|n\right\rangle \left\langle n \left|\mathbf{E}_{o}\right\rangle\right.}{1 - w_{n}} \tag{2}$$

where  $|n\rangle$  and  $w_n$  are the eigenvectors and eigenvalues respectively of the total interaction matrix.  $|\mathbf{E}_0\rangle$  represents the incident field. These vectors have all 3N elements and contain the oscillation modes or the incident electric field on each particle.

#### **3.2** Particles with $\mu \neq 1$

In the previous section we saw that the coupling between two electric dipoles produces four types of modes: transversal and longitudinal modes, where both of them have symmetric and anti-symmetric variants. If we now introduce magnetic dipoles ( $\mu \neq 1$ ), new modes appear. We still have the purely electric longitudinal modes, but now we have also their purely magnetic counterpart. The transversal modes appear now as mixed electric and magnetic states.

As an example, we will choose here  $\mu = -1.6$ . The scattering cross section as a function of interparticle distance is represented in figure 2(a). We observe similarities but also differences between this case and the previous one with  $\mu = 1$ . The peak centred in  $r/\lambda = 1.5$  is now the most important. As can be seen from figure 2(a), the mode corresponding with interparticle distance  $r/\lambda = 1.5$  has anti-symmetrical electric and symmetric magnetic components.



Figure 2: (a) Scattering cross-section  $\sigma_s$  as a function of the interparticle distance for  $\varepsilon = -2.166$  and  $\mu = -1.6$ . (b) LPC obtained for totally unpolarized incident radiation.

In the LPC plot we see for  $r/\lambda = 1.5$  a strong minimum with negative values, which indicates a negative polarization branch. This plot was obtained for a scattering angle of  $\theta_s = 150^\circ$  which is the direction where the minimum of the NPB usually occurs. The same happens for  $r/\lambda = 1.0$ . In  $r/\lambda = 0.7$ , the linear

polarization doesn't become negative which indicates again a qualitative difference between the resonances.

### **4** Conclusions

We studied the resonance spectra and the linear polarization coefficient of a Dipolar Bi-Sphere as a function of the interparticle distance where the constituents can be considered magnetic or non-magnetic. Depending on the type of excited resonance, we can or cannot produce a negative polarization branch. In the case of non-magnetic particles with  $r/\lambda = 1.0$ , we found that both the transversal symmetrical and anti-symmetrical states do contribute to the negative polarization branch.

For magnetic particles the highest peak was found for  $r/\lambda = 1.5$  corresponding with an anti-symmetric electric and symmetric magnetic mode and produces NPB.

These results point out that not only the presence of multiple scattering is important, but that the oscillation mode plays also a fundamental role.

### Acknowledgements

The authors wish to thank the Dirección General de Enseñanza Superior for its financial support (FIS2004-06785). Olivier Merchiers wishes to thank the University of Cantabria for his research grant.

## References

- G. Videen, Y. Yatskiv, and M. Mishchenko, eds., Photopolarimetry in Remote Sensing, vol. 161 of NATO Sci Series, Kluwer, Dordrecht, 2004.
- Y. Shkuratov, A. Ovcharenko, E. Zubko, O. Miloslavskaya, K. Muinonen, J. Piironen, R. Nelson, W. Smythe, V. Rosenbush, and P. Helfenstein, Icarus 159, 396-416 (2002).
- [3] Y. Shkuratov, A. Ovcharenko, E. Zubko, H. Volten, O. Muñoz, and G. Videen, J. Quant. Spectr. Rad. Transf. 88, 267-284 (2004).
- [4] V.A. Markel, Journal of Modern Optics **39**(4), 853-861 (1991).
- [5] F. Ismagilov and Y. Kravtsov, Waves in Random Media **3**, 17-24 (1993).
- [6] F. Moreno, O. Merchiers, and F. González, Optics Letters 30, 2194-2196 (2005).
- [7] G. Dolling, M. Wegener, C. M. Soukoulis and S. Linden, Optics Letters 32, 53-55 (2007)
- [8] M. Kerker, D.-S. Wang, and C.L. Giles, JOSA 3, 765-767 (1982)
- [9] S. Singham and C. Bohren, J. Opt. Soc. Am. A 5 (11), 1867-1872 (1988).
- [10] G.W. Mulholland, C.F. Bohren, and K.A. Fuller, Langmuir 10 (8), 2533-2546 (1994).