LAMINAR-TO-TURBULENT FLOW TRANSITION IN MICRO-CHANNELS

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Some experimental results on heat transfer and pressure drop through micro-channels evidenced that the classical theory, valid for large sized channels, could be not valid for channels having an hydraulic diameter smaller than 1 mm. Also the transition from laminar to turbulent regime is found to happen to Reynolds number lower than the expected value and this fact is considered a confirmation of the disagreement with the classical theory; in this paper is shown as some experimental results on the transition from laminar to turbulent in micro-channels can be explained by using the "Obot-Jones" model for large sized channels.

INTRODUCTION

The development of micro-fluidics devices has been particularly striking during the past 10 years. Today, the research on MEMS (Micro-Electro-Mechanical Systems) is exploring different applications which involve the dynamics of fluids and the single and two-phase forced convective heat transfer. In some cases the micro-channels are produced directly by etching on the silicon wafers; in these devices, the shape of the channels depends on a variety of factors such as the crystallographic nature of the silicon used. For example, when a photo-lithographic technique is employed, the micro-channels etched in <100> or in <110> silicon using a KOH solution, will have a trapezoidal cross-section (with a sharp angle of 54.74° imposed by the crystallographic morphology of the silicon) or a rectangular cross-section respectively. In the present paper, KOH-etched micro-channels having cross-sections trapezoidal, hexagonal (obtained gluing together two trapezoidal channels) and rectangular will be considered.
An interesting aspect of fluid dynamics through micro-channels is lied to the transition from laminar to turbulent regime. Some studies indicate that the transition from laminar to turbulent flow in micro-scale passages takes place at "critical" Reynolds number ranging from 300 to 2000. In particular, the experimental data of Wu and Little[1] on trapezoidal glass and silicon micro-channels, indicated that for $\mathrm{Re}<1000$ the flow is laminar, for $1000<\mathrm{Re}<3000$ the flow drops into the transition region and for $\mathrm{Re}>3000$ the flow is fully turbulent. Choi *et al.*[2], analyzing microtubes with an hydraulic diameter of 53 μm and 81.2 μm, indicated that the transition to turbulent flow occurs at Re=2000. They found that this value decreases for micro-channels having an hydraulic diameter smaller (Re=500 for D_h=9.7 μm and 6.9 μm). The experimental analysis of metallic micro-channels conducted by Peng *et al.*[3-4] indicated that the critical Reynolds values for the flow regimes through rectangular micro-channels could be less than the values found by Wu and Little; Peng and Peterson[5] indicated that the $\mathrm{Re}\in[200\text{-}700]$ range represents the upper bond for laminar flow transition to turbulence. In particular, Peng and Peterson gave $\mathrm{Re}<400$ for laminar flow, $400<\mathrm{Re}<1000$ for the transition region and $\mathrm{Re}>1000$ for fully turbulent flow. Harms *et al.*[6] found that for deep rectangular micro-channels having an aspect ratio of 0.244 the critical Reynolds number is about 1500. Obot[7], analyzing critically the experimental works cited above, arrives to the conclusion that there is not strong experimental evidence to support the existence of transitional or turbulent flow for Reynolds number less than 1000. In addition, he underlines that there are misconceptions on the transition region in the literature on micro-channels.

THE “OBOT-JONES” MODEL

In this paper the “Obot-Jones” method for conventional sized channels will be utilized in order to predict the value of the critical Reynolds number that corresponds to the onset of the transition from laminar regime and turbulent regime. The role of the cross-section geometry, of the aspect ratio and of the relative roughness (e/D_h) on the transition will be underlined. Jones[8] noted that, for non-circular duct, at constant Reynolds number, based on hydraulic diameter, the friction factor increases monotonically with increasing aspect ratio. He concluded that the hydraulic diameter is not the proper length dimension to use in the Reynolds number to insure similarity between the circular and non-circular ducts. For this reason, Jones introduced a modified characteristic length (named “laminar equivalent” diameter) defined as follows:

$$D_L = \Phi^* D_h \tag{1}$$

where the geometry function Φ^* depends only, for fixed configuration of the channel cross section, on the aspect ratio γ. As shown by Jones for rectangular duct, for fixed geometry of the channel, it is possible to calculate Φ^* by means of the fully developed Poiseuille number for laminar regime. Obot[9] developed the Critical Friction Method (CFM) in order to reduce the friction data for non-circular passages on the classical circular tube relations. The CFM method indicated that Φ^* is related to the ratio between the critical Reynolds number for circular ($Re_{c,c}$) and non circular ducts ($Re_{n,c}$). By using this method, the determination of the value of the critical Reynolds number for a micro-channel having any cross-section geometry can be calculated by means of the following equation:

$$\mathrm{Re}_{n,c} = \frac{\mathrm{Re}_{c,c}}{\Phi^*} \tag{2}$$

By Eq.(2) it is possible to note how, for non-circular channels, the transition between the laminar and turbulent flow regime could happen to critical Reynolds values less than for circular ducts (if Φ^* greater than 1). In Fig. 1 the value of Φ^* is quoted for trapezoidal, rectangular and double-trapezoidal KOH-etched micro-channels as function of the aspect ratio (γ=max heigth/ max width). It is possible to note that Φ^* is less than 1 for $\gamma<0.268$ for trapezoidal channels, for $\gamma<0.489$ for hexagonal channels and for $\gamma<0.441$ (or $\gamma>2.268$) for rectangular channels.

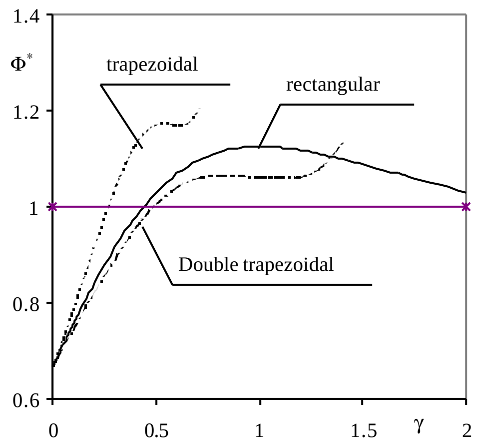

*Fig. 1 – The parameter F * defined by Eq.(1) as function of the micro-channel cross-section geometry.*

The value of the critical Reynolds number for circular duct $Re_{c,c}$ that is used in Eq.(2) can be calculated by using the well-known relations for rough tubes. For instance, Celata *et al.*[10], in order to predict the critical Reynolds number for a rough capillary tube, used the model of Preger and Samoilenko, quoted in Idelchick[11]. In this manner, it is possible to underline the role of the relative roughness on the transition from laminar to turbulent regime; for high relative roughness the critical Reynolds number decreases. This fact can partially explain the results of some investigators that obtained an "early" laminar-to-turbulent transition through rough micro-channels.

CONCLUSIONS

The "Obot-Jones" model permits to understand the role of the cross-section geometry and of the wall roughness on the transition from laminar to turbulent flow through micro-channels.
Even if the "Obot-Jones" model is valid for large sized channels, a comparison between the model and the experimental data obtained by Wu and Little[1], Acosta *et al.*[12] and Harms *et al.*[7] permits to verify that the model is suitable for micro-channels with an hydraulic diameter greater than 40 μm in order to predict the laminar-to-turbulent flow transition.

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