GALERKIN METHOD IN TRANSIENT RADIATIVE TRANSFER

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Galerkin technique is employed for the solution of radiative transfer in a one-dimensional absorbing and isotropically scattering plane-parallel gray medium with short-pulse irradiation on one of its boundaries. Two different approaches are included: Direct application of the Galerkin method and a Laguerre-Galerkin approach. The transient transmittance and refectance of the medium are evaluated for various values of the optical thickness, scattering albedo and pulse duration. The Laguerre-Galerkin method is demonstrated to be easier to implement and more efficient when compared to the direct application of the Galerkin method.

INTRODUCTION

The problem of transient radiative transfer in participating media has attracted rising interest due to the recent applications that involve extremely small time scales. In classical radiation problems, the time derivative term in the radiative transfer equation has a negligible order of magnitude compared to the others. Lasers of pico to femto-second pulse durations are now available. Short-pulse lasers are being used to investigate the properties of scattering and absorbing media in such applications as, optical tomography, combustion product analysis, and remote sensing. For such applications, the time derivative in the radiative transfer equation can no longer be neglected. Numerous approaches such as, integral formulation, direct numerical approach, discrete ordinates method, and Monte Carlo simulations have been introduced for the solution of transient radiative transfer problems in participating media. In the present work, direct Galerkin[1] and Laguerre-Galerkin[2] solutions are presented.

SOLUTION PROCEDURE

A one-dimensional, absorbing, isotropically scattering, plane parallel gray medium with non-reflecting and non-refracting boundaries is considered as shown in Figure 1. The medium is exposed to irradiation of Gaussian profile on the boundary at $x = 0$.

In the direct application of the Galerkin method, the integral form of the radiative transfer equation for the time-dependent source function is transformed into a set of ordinary differential equations for the time-dependent expansion coefficients of the power series expansion of the source function. The time derivative in the radiative transfer equation is replaced by a backward finite difference scheme to solve the set of ordinary differential equations numerically over the time spectrum.

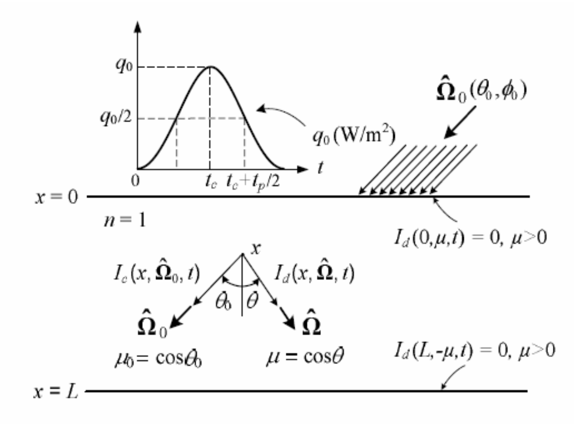

Figure 1. Schematic representation of boundary conditions.

In Laguerre-Galerkin method the time-dependent dimensionless radiative intensity within the medium is expanded in a series of Laguerre polynomials with time as the argument. The moments of the radiative transfer equation, as well as of the boundary conditions, are then taken in accordance with the orthogonality property of the Laguerre polynomials, yielding a set of coupled time-independent radiative transfer problems. This set, in turn, is reduced to a set of algebraic equations utilizing the Galerkin method.

RESULTS

The direct Galerkin results are compared with those obtained by the integral equation solution (IES), the MDA and the modified $P_{1/3}$ approximation ($MP_{1/3}A$) methods. Figures 2 and 3 compare the reflectance and transmittance results, respectively, for $\tau_0 = 2$, $t_p^* = 0.3$, $t_c^* = 3t_p^*$ and $\omega = 1$. Reflectance results show very good agreement with those available in the literature. Transmittance curves obtained by the direct Galerkin method also show similar trends. It is also observed that the convergence of our transmittance results is faster for dimensionless pulse durations at and above 1.

The Laguerre-Galerkin results are compared to those obtained by the direct application of the Galerkin method. Figures 4 and 5 compare the reflectance and the transmittance results, respectively, for $\tau_0 = 1$. It should be noted that this corresponds to an optical thickness of 0.5 for the direct application of the Galerkin method due to the difference in defining the dimensionless parameters. Both transmittance and reflectance results show excellent agreement with those obtained by the Galerkin method.

CONCLUSION

Galerkin and Laguerre Galerkin techniques have been developed for transient radiative transfer in participating media exposed to collimated short-pulse Gaussian irradiation. The problem has been formulated for a non-emitting, absorbing and scattering plane parallel gray medium with non-reflecting and non-refracting boundaries. The effects of the optical thickness, scattering albedo and pulse duration on transient transmittance and reflectance have been investigated. The results agree well with those available in the literature. Utilizing the Laguerre-Galerkin technique, the transient transmittance and refectance are re-evaluated.

An excellent agreement has been reached with previously obtained results. The Laguerre-Galerkin approach is more efficient compared to the direct application of the Galerkin method. Both methods work better for small to moderate optical thicknesses where the hyperbolic approximations reported in the literature seem to fail.

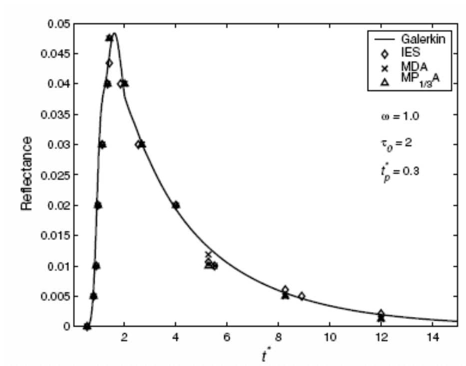

Figure 2. Comparison of transient reflectance results obtained by Galerkin, IES, MDA and $MP_{1/3}A$ methods for $\tau_0 = 2$, $t_p^* = 0.3$, $t_c^* = 0.9$ and $\omega = 1$.

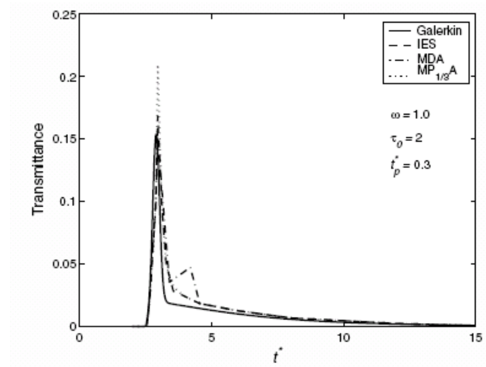

Figure 3. Comparison of transient transmittance results obtained by Galerkin, IES, MDA and $MP_{1/3}A$ methods for $\tau_0 = 2$, $t_p^* = 0.3$, $t_c^* = 0.9$ and $\omega = 1$.

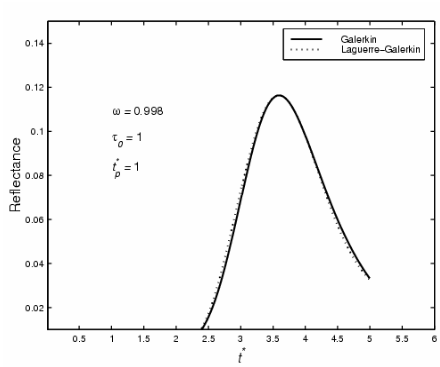

Figure 4. Comparison of transient reflectance results obtained by Laguerre-Galerkin ($\tau_0 = 1$) and Galerkin ($\tau_0 = 0.5$) methods for $t_p^* = 1$ and $\omega = 0.998$.

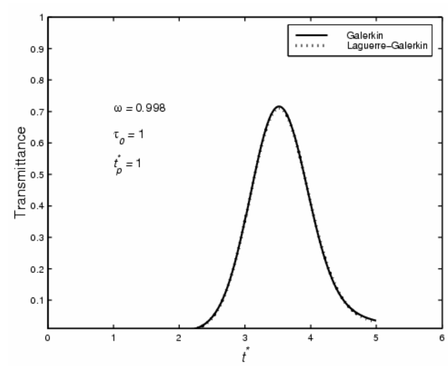

Figure 5. Comparison of transient transmittance results obtained by Laguerre-Galerkin ($\tau_0 = 1$) and Galerkin ($\tau_0 = 0.5$) methods for $t_p^* = 1$ and $\omega = 0.998$.

REFERENCES

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