Spatio-temporal identification of a heat source in a diffusive by solving a reverse problem

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Abstract:
In this work, we present a numerical study to determine a heat source density, homogeneous in a domain. We show how a point source "equivalent" to approach the phenomenon. The method is based on a technique of inversion of a convolution integral, and secondly, on a direct modeling of the system by finite differences.

Nomenclature:

a: thermal diffusivity, $m^2.s^{-1}$
c: heat capacity, J. $kg^{-1}.k^{-1}$
e_S, e_T: root mean square deviation, ° C
F: final time, s
h: convection coefficient, $W.m^{-2}.K^{-1}$
H (t): impulse responses
M (t): answers index
t: time, s
T: temperature, K
u (t), y (t): vector
xc, yc: coordinates of the source, m

Greek symbols
Δ: Laplacian operator
λ: thermal conductivity, $W.m^{-1}.k^{-1}$
ρ: density, $kg.m^{-3}$
τ: time, s

Indices
EPS: equivalent source
i, j, k, n, m: indices

1. Introduction

We deal in this work a problem of identification of a volume heat source in a diffusive 2D, the source is assumed uniform. We start first by looking for its position is determined by its amplitude as a function of time.

To address this problem, we use a direct modeling by finite difference, to store the simulated temperatures. The other way round it is based on an inversion of the integral of convolution type Beck [1]. In this phase inversion of the system is considered under the aspect monoentrée (the source to be identified) multisorties (data from internal and surface temperatures). The source density 2D we use to simulate our temperature measurements is circular.

1.1. Inversion method :

1.1.1. Integral Convolution

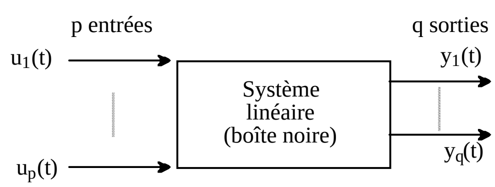

Figure 1 *: Principe de la représentation externe*

Knowing the matrix H (t), then it is conventional to write the system's response to any vector u (t) in the form of the convolution integral [2].

$$y(t) = \int_{t_0}^{t} H(t-\tau)u(\tau)d\tau \qquad (1)$$

where u (t) is the vector of input dimension (p), y (t) the output vector of dimension (q), H (t) the impulse response matrix of dimension (qxp), t is time, t0 the initial time for which y (t0) = 0. Using in equation (1) the matrix of responses index M (t) it comes:

$$y(t) = \int_{t_0}^{t} -\frac{dM(t-\tau)}{d\tau} u(T)d(T) \qquad (2)$$

Δt is the constant time step. By putting t = F Δt l'instant calculation and t= f Δt current time integration, a finite difference approximation to the first order derivative of M (t), it comes:

$$y(F) = \sum_{f=1}^{F} \Delta M_{F-f} u(f) \qquad (3)$$
$$\text{avec} \quad \Delta M_{F-f} = M(F-f+1) - M(F-f)$$

To determine the vector u entries F at the moment, we will use equation (3) extended to the future time step. Explain this equation by writing it at the time of calculating F:

$$y(F) = \Delta M_{F-1}\, u(1) + \Delta M_{F-2}\, u(2) + \ldots + \Delta M_1\, u\,(F-1) + \Delta M_0\, u\,(F) \qquad (4)$$

In this expression u (1),... u (F-1) are known, u (F) is unknown. We may combine in a term y * (F) any contribution of the moments prior to F in the form:

$$y^*(F) = \Delta M_{F-1}\, u(1) + \Delta M_{F-2}\, u(2) + \ldots + \Delta M_1\, u\,(F-1) \qquad (5)$$

Equation (4) daemon:

$$y(F) - y^*(F) = \Delta M_0\, u(F \qquad (6)$$

Or in matrix form: $\quad Y = M\, u(F) \qquad (7)$

The relationship (7) constitutes a system of q * (R +1) equations with p unknowns. In general this system is over determined, because q * (R +1)> p. The exact resolution is not possible, by choosing a standard quadratic, it is then the solution of equation (7) in the sense of least squares, either:

$$u(F) = (M^T M)^{-1} M^T Y \qquad (8)$$

The inverse problem is solved at each time step F, taking into account R future time steps, by the relation (8).

1.2. The direct model finished in deference:

The finite difference equations can be established in two ways, either by using the results of numerical analysis, or by writing the bilant thermal each node of the network. The second method allows a more physical problem [3].

$$\Delta T - \frac{1}{a}\frac{\partial T}{\partial t} + \frac{P(M,t)}{\lambda} = 0 \quad (9)$$

$$T_{i,j}^{t+\Delta t} = c_i T_{i,j} + a_i\left(T_{i+1,j} + T_{i-1,j}\right) + b_i\left(T_{i,j+1} + T_{i,j-1}\right) + P_{i,j}\frac{a\Delta t}{\lambda \Delta x \Delta y} \quad (10)$$

$$T_{n,j}^{t+\Delta t} = T_{n,j}\left(1 - 2a_i - 2b_i - \frac{2ah\Delta t}{\lambda \Delta y}\right) + a_i\left(T_{n,j-1} + T_{n,j+1}\right) + 2b_i T_{n-1,j} + \frac{2ah\Delta t}{\lambda \Delta y}T_\infty \quad (11)$$

$$T_{1,j}^{t+\Delta t} = T_{1,j}\left(1 - 2a_i - 2b_i - \frac{2ah\Delta t}{\lambda \Delta y}\right) + a_i\left(T_{1,j-1} + T_{1,j+1}\right) + 2b_i T_{2,j} + \frac{2ah\Delta t}{\lambda \Delta y}T_\infty \quad (12)$$

$$T_{i,m}^{t+\Delta t} = T_{i,m}\left(1 - 2a_i - 2b_i - \frac{2ah\Delta t}{\lambda \Delta x}\right) + b_i\left(T_{i+1,m} + T_{i-1,m}\right) + 2a_i T_{i,m-1} + \frac{2ah\Delta t}{\lambda \Delta x}T_\infty + P_{1,m}\frac{2\Delta t}{\rho c \Delta x \Delta y} \quad (13)$$

$$T_{i,1}^{t+\Delta t} = T_{i,1}\left(1 - 2a_i - 2b_i - \frac{2ah\Delta t}{\lambda \Delta x}\right) + b_i\left(T_{i+1,1} + T_{i-1,1}\right) + 2a_i T_{i,2} + \frac{2ah\Delta t}{\lambda \Delta x}T_\infty + P_{i,1}\frac{2\Delta t}{\rho c \Delta x \Delta y} \quad (14)$$

$$T_{n,1}^{t+\Delta t} = T_{n,1}\left(1 - 2a_i - 2b_i - 2\frac{ha\Delta t(\Delta x + \Delta y)}{\lambda \Delta x \Delta y}\right) + 2a_i T_{n,2} + 2b_i T_{n-1,1} + 2\frac{ha\Delta t(\Delta x + \Delta y)T_\infty}{\lambda \Delta x \Delta y} \quad (15)$$

$$T_{n,m}^{t+\Delta t} = T_{n,m}\left(1 - 2a_i - 2b_i - 2\frac{ha\Delta t(\Delta x + \Delta y)}{\lambda \Delta x \Delta y}\right) + 2a_i T_{n,m-1} + 2b_i T_{n-1,m} + 2\frac{ha\Delta t(\Delta x + \Delta y)T_\infty}{\lambda \Delta x \Delta y} \quad (16)$$

$$T_{1,m}^{t+\Delta t} = T_{1,m}\left(1 - 2a_i - 2b_i - 2\frac{ha\Delta t(\Delta x + \Delta y)}{\lambda \Delta x \Delta y}\right) + 2a_i T_{1,m-1} + 2b_i T_{2,m} + 2\frac{ha\Delta t(\Delta x + \Delta y)T_\infty}{\lambda \Delta x \Delta y} \quad (17)$$

$$T_{1,1}^{t+\Delta t} = T_{1,1}\left(1 - 2a_i - 2b_i - 2\frac{ha\Delta t(\Delta x + \Delta y)}{\lambda \Delta x \Delta y}\right) + 2a_i T_{1,2} + 2b_i T_{2,1} + 2\frac{ha\Delta t(\Delta x + \Delta y)T_\infty}{\lambda \Delta x \Delta y} + P_{1,1}\frac{4\Delta t}{\rho c \Delta x \Delta y} \quad (18)$$

Avec : $a_i = \frac{a\Delta t}{\Delta x^2}$, $b_i = \frac{a\Delta t}{\Delta y^2}$ *et* $c_i = 1 - \frac{2a\Delta t}{\Delta x^2} - \frac{2a\Delta t}{\Delta y^2}$

2. The system studied and its modeling :

In Figure 3 are represented the various elements of the problem:

❖ Geometry studied is a flat rectangular plate of length 0.2 m and a width of 0.1 m. The thermo- physical characteristics are selected for a building material like plaster. Values [4]:

- Thermal conductivity $\lambda = 1.5$ $Wm^{-1}.k^{-1}$
- Density $\rho = 2300$ $kg.m^{-3}$
- Heat capacity $c = 800$ $J.kg^{-1}.k^{-1}$

❖ On all 4 sides of the rectangle, the boundary conditions of the field are Fourier type (h = 20 W m-2 K-1) in relation to a reference temperature taken as 0 ° C. At the initial moment, the whole system is at 0 ° C [4].

❖ The circular source centered at point C: xc = 5 10-2 m, yc 6 = 10-2 m and radius R = 2.5 10-2 m.

❖ To model this system in deference over, you have a mesh of the plate ΔY = 0005 m and Δ X = 0.01 m:

Let P (t) the power emitted by this source as a function of time (see Figure 4).

3- Location method (method of removal of the domain):

The idea is based on an iterative method, presented by the following algorithm [4] (see Figure 2):

1) The plate (with dimensions: xI = 0.2 m, yl = 0.1 m) is separated into two (2) A and B at its center, by a parallel to Oy.
2) Two sources fictitious SP-A and SP-B are placed in each of the points A and B, the abscissa $x_a = 3 / 8$ xI and $x_b = 5 / 8$ xI and ya = yb = yg = 1 / 2 yl.
3) Recovery of simulated temperatures.
4) Calculation of index responses for each source (SP-A and SP-B) on the first step (construction of the matrix M of (3).
5) Calculation of the vector u (1) by the relation (8) on the first step which is:
$$u(1) = (M^T.M)^{-1} M^T Y(R).$$
The vector u (1) has two components ua (1) and ub (1) with respect to SP-A and SP-B.
6) A center of gravity between SP-A and SP-B will be calculated.
7) The part not containing the center of gravity is "disposed" of the algorithm.
8) It works in the opposite direction on a parallel to Oy via xg creation of SP-A and SP-B following xa = xb = xg, where $y_a = 3 / 8$ and yb= 5 / 8 yl. Reversing the x and y and on again in 2 until convergence (Figure 5).
9) The final solution, ie the position of the source, is the center of gravity when it almost does not move during the iterations.

Once the source is located on the first step, there is a direct simulation:
it makes a step on this source over the length of the interval time of study. Replies index, the measurement points, are stored (for the construction of the matrix M) is calculated by the source value at each time step, from the relationship (8).

3. Result:

The location is therefore searching for a point source "equivalent" (SPE), successive iterations can identify the center of gravity G, the position taken by the PES, the "epicenter" of the source is located fairly accurately.

In Figure 5, it represents the convergence of the position of G according to the number of iterations, you can see how successive calculations of xg and yg tend towards the values xc and yc sought. SPE is localized, then we apply the inversion method on the whole time horizon. We present in Figure 6, the source value compared with the SPE theoretical curve used for the direct model and the quadratic differential on sources eS (W / m). We note that levels are not well reproduced despite the best location, the reason that the two sources are not similar because here we identify a point source equivalent.

In Figure 7 presents examples of thermogram reconstructed from identification with the root mean square deviation and (in ° C), we see a good agreement until the time t = 4000s, after this time is a little less and a better fit for the curves of temperatures that are far from the source.

4. Conclusion

In this work, we presented a method of inversion in thermal conduction, to identify a volume heat source in a diffusive 2D. This inverse problem has two steps to locate the position of this source, then an identification of its amplitude as a function of time. So on the heat source density, we notice that we have a good result for the location, whilst the intensity of the source volume we have a worse outcome, because we identified a point source is equivalent to an approximate solution, but the final solution is still the identification of the geometric shape of this source.

References

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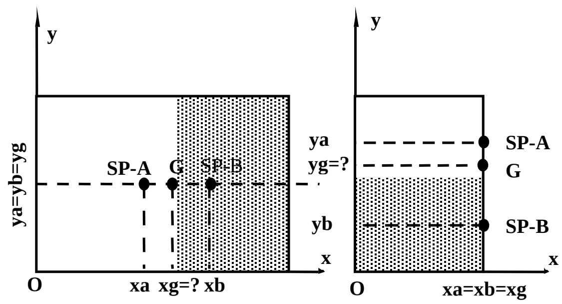

Figure 2: Principle of localization, elimination of field

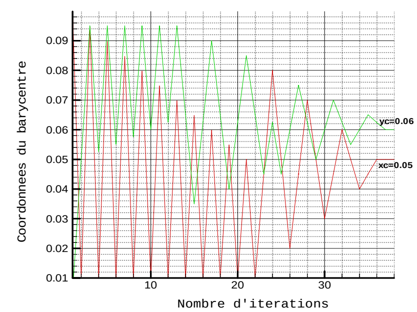

Figure 5: Convergence of center of gravity

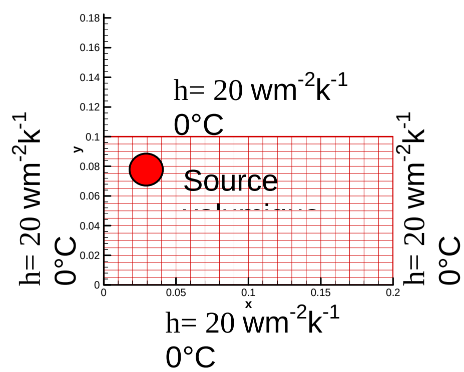

Figure 3: The test case studied with these conditions

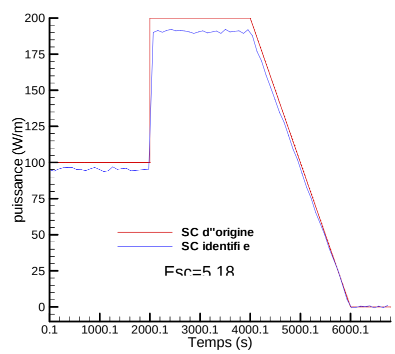

Figure 6: Comparison of the circular source of origin and the point source equivalent (SPE)